

A HYBRID APPROACH COMBINING CONJOINT ANALYSIS AND THE ANALYTIC HIERARCHY PROCESS FOR MULTICRITERIA GROUP DECISION-MAKING

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ABSTRACT

In this article, we introduce the Conjoint Analytic Hierarchy Process (CAHP), a novel multi-criteria aggregation function that hybridizes Conjoint Analysis (CA) and the Analytic Hierarchy Process (AHP). Most of the limitations of traditional multi-criteria methods are addressed by CAHP. The proposed approach has many practical implications in various sectors such as business, industry, healthcare, education, and more. The keystone of the method is to apply CA to obtain the weights of criteria before

applying the usual AHP in the subsequent steps (level of alternatives). Prior to using the AHP, decision tables from decision-makers were transformed into a unique decision table using the arithmetic mean of alternatives' performances on criteria. Appropriate formulas were then used to turn this aggregated decision table into pairwise comparison matrices, upon which the AHP was applied. We tested the CAHP in two real-world situations to demonstrate its reliability. The results show that the rankings obtained from CAHP are identical to those from other methods, such as TOPSIS, ELECTRE II, and PROMETHEE II. Future research should focus on developing user-friendly tools to facilitate CAHP application. Other perspectives would involve carefully classifying each criterion's modalities to prevent inversions in respondent preferences during CA and assessing possible biases to manage unexpected preferences.

Keywords: AHP; Conjoint Analysis; criteria weighting; group decision-making; multicriteria analysis; pairwise comparison

1. Introduction

Making decisions is a fundamental characteristic of everyday human life and each choice has a specific objective. In this decision-making process, most of the time, people have to face complex scenarios where they must consider several points of view (Mousseau et al., 2000) which aim to reconcile multiple and interdependent benefits and risks (Taherdoost & Madanchian, 2023). Indeed, each choice – whether individual or collective – involves identifying and selecting alternatives depending on multiple, often conflicting criteria that must be taken in account (Roy, 1968). Such challenges are referred to as Multi-Criteria Decision-Making (MCDM) problems.

In the realm of MCDM, we find a multitude of aggregating functions that aid in making rational decisions. Among these, the Analytic Hierarchy Process (AHP) and Conjoint Analysis (CA) stand out as powerful tools for prioritizing criteria and optimizing choices, thus maximizing the probability of an informed and rational decision. The AHP is a structured technique inspired by mathematics and psychology, introduced by Saaty (1980). It involves decomposing a decision problem into a hierarchy of sub-problems, which can be analyzed independently. CA is a statistical technique used in the marketing domain to analyze how people value different attributes of a product or service.

In this article, we aim to hybridize both techniques to extend the AHP so that it can address multi-criteria problems involving multiple decision-makers, calling it the Conjoint Analytic Hierarchy Process (CAHP). Such an approach has been developed by many other researchers in the field of MCDM. Among others, Beynon et al. (2000) combined the Dempster-Shafer theory of evidence with the AHP; Soltanifar & Kamyabi (2024) developed the Voting AHP (VAHP), which combines a voting function with the AHP; Zolfani & Saparauskas (2013) hybridized the AHP with SWARA to evaluate alternatives by prioritizing expert criteria and initiating the weighting process to determine relative weights; Rezaei (2015) proposed a method combining the AHP and the Best-Worst Method (BWM); Mi et al. (2019) suggested hybridizing the BWM and the VAHP to address multi-criteria problems.

Although these methods help find solutions to MCDM problems, they are subject to fundamental limitations such as increasing complexity, lack of consistency in judgments,

oversensitivity to changes in weights, and more. In this study, we argue that CAHP addresses these drawbacks and helps find rational solutions to multi-criteria problems with multiple decision-makers.

2. Literature review

2.1 Multicriteria Decision-Making

Multi-criteria decision-making, also referred to as multi-attribute decision-making, involves selecting the best alternative from a finite set of options. In most complex decision-making problems, the evaluation of alternatives is based on multiple criteria, which are often in conflict with one another. In such cases, it may be impossible to identify an alternative that simultaneously satisfies all criteria (Jahanshahloo et al., 2009). According to Roszkowska (2011), in a multi-criteria decision-making process, the decision-maker must:

- Define the criteria based on the objectives;
- Generate alternatives to achieve the objectives;
- Evaluate the alternatives in terms of the criteria;
- Apply a multi-criteria analysis method to determine the best alternative. If the final solution is not accepted, gather new information to restart the decision-making process.

Multi-criteria decision-making, primarily based on issues of choice, sorting, and ranking, employs various methods, among which the AHP is one of the most frequently used for rational decision-making. To solve a multi-criteria decision-making problem, the decision-maker constructs a decision matrix in which the performances of the alternatives with respect to each criterion can be represented by real numbers, intervals, fuzzy numbers, or qualitative labels (Roszkowska, 2011).

For an identified multi-criteria decision-making problem, let us denote:

- $A = \{a_1, a_2, \dots, a_n\}$ the set of n alternatives ;
- $C = \{c_1, c_2, \dots, c_m\}$ the set of m criteria that influence the choice of the decision-maker;
- $W = (w_1, w_2, \dots, w_m)$ a vector where $w_j \in \mathbb{R}_0^+$ ($1 \leq j \leq m$) is the weight or importance that the decision-maker assigns to criterion c_j and $\sum_{j=1}^m w_j = 1$;
- $P = \{p_{ij}/i = 1, \dots, n; j = 1, \dots, m\}$ the set of performances of alternative a_i on criterion c_j .

Thus, the multi-criteria decision-making problem can be succinctly illustrated in the form of a matrix, as shown in Table 1.

Table 1
Multi-criteria decision-making matrix

Criteria	c_1	c_2	...	c_m
Weights	w_1	w_2	...	w_m
a_1	p_{11}	p_{12}	...	p_{1m}
a_2	p_{21}	p_{22}	...	p_{2m}
...	...	...	...	...
a_n	p_{n1}	p_{n2}	...	p_{nm}

2.2 Multicriteria problems with multiple decision-makers

Due to the complexity of issues addressed in various sectors of life, it is difficult, if not impossible, for a single individual to make decisions. In such cases, the decision-making process involves multiple decision-makers, which is referred to as group decision-making. Each decision-maker participating in this process may make decisions based on their own motivations or objectives, but with a shared interest in reaching a final agreement on the selection of the 'best' alternative(s) (Meng & Chen, 2015). To this end, decision-makers must express their preferences through a set of evaluations on a range of possible alternatives (Herrera-Viedma et al., 2007).

Thus, for a multi-criteria multiple decision-making problem, each decision-maker d_k (where $k = 1, \dots, l$) evaluates a set of n alternatives a_i (where $i = 1, \dots, n$) based on m criteria c_j (where $j = 1, \dots, m$). Let p_{ijk} and w_{jk} represent, respectively, the performance of alternative a_i on criterion c_j and the weight of criterion c_j , both assigned by decision-maker d_k .

2.3 Conjoint Analysis (CA)

In the early 1970s, Green and Srinivasan (1971) introduced CA to identify customer preferences for a product or service. This technique uses a decomposition approach to evaluate the value of various attribute levels based on respondents' assessments of hypothetical profiles (Kuzmanovic & Savic, 2020). The first review of CA was conducted a couple of years later (Green & Srinivasan, 1978) before being updated and extended in the early 1990s. Since its proposal, CA has garnered significant interest from both academia and industry as a crucial set of methodologies for assessing buyers' preferences and trade-offs among products and services with multiple attributes (Green & Srinivasan, 1990). As shown in Table 2, CA can be applied in different fields.

Table 2
Recent articles on CA technique

Authors	Application Area	Specific Objective	Tools Used
Lou & Xu (2024)	Sustainable denim products	Contribution of blockchain certified eco-labels	Conjoint Analysis
Zhang et al. (2024)	Carbon tax pricing	Evaluate preferences of Shanghai residents for carbon tax policies	Conjoint Analysis, Choice-based Conjoint Analysis Method
Hong et al. (2024)	Green tea products	Analyze consumer preferences for green tea products in Thai Nguyen, Vietnam	Conjoint Analysis
Chang et al. (2022)	Sustainable luxury consumers	Analyze sustainable luxury consumers' preferences and segments	Conjoint Analysis, Cluster Analysis
Wang et al. (2022)	Yogurt packaging design	Analyze consumer preferences for yogurt packaging design	Conjoint Analysis

CA involves three steps:

1. Preference measurement: Preferences are measured via ranking or rating tasks. The relative importance I_k of attribute k is given by Equation (1):

$$I_k = \frac{\max(u_{kj}) - \min(u_{kj})}{\sum_k (\max(u_{kj}) - \min(u_{kj}))} \quad (1)$$

Where u_{kj} is the utility of the level j of the attribute k .

2. Utility estimation: Utilities u_{kj} are estimated using models like MONANOVA, LINMAP, OLS, LOGIT, or PROBIT. For linear models, Equation (2) provides the utilities:

3.

$$u_{kj} = \beta_{kj} \cdot x_{kj} \quad (2)$$

Where β_{kj} is the parameter estimate for level j of attribute k .

4. Experimental design: Fractional factorial designs (e.g., Latin squares) reduce the number of profiles. For three attributes A, B, C, each with three levels, a Latin square reduces 27 profiles to 9.

2.4 Analytic Hierarchy Process (AHP)

The AHP was first introduced by Saaty (1980). The earliest reference to this method can be traced back to his earlier work (Saaty, 1972). In the late 1970s, an article published in the *Journal of Mathematical Psychology* provided a precise description of the AHP

(Saaty, 1977). The AHP is a structured methodology for organizing and analyzing complex decisions and is also one of the most used multi-criteria methods in the decision-making process (see Table 3). It involves decomposing a decision problem into a hierarchy of criteria, sub-criteria, and alternatives. Pairwise comparisons are used to obtain priority weights for each element (criteria, sub-criteria, or alternatives) in the hierarchy. The decision is decomposed into hierarchical levels. The hierarchy consists of a goal (top level), a set of m criteria (intermediate level), and a set of n alternatives (lowest level). Each criterion c_j may further be decomposed into sub-criteria if necessary.

Table 3
Recent articles on the AHP approach

Authors	Application Area	Specific Objective	Tools Used
Nguyen et al. (2023)	Manufacturing	Develop indicators	Delphi & AHP
Madzik & Falt (2022)	Decision-making	Review AHP	LDA Topic Modelling
Amenta, et al.(2021)	Group decision-making	Weights aggregation	Frobenius Norm Algorithm
Aguarón et al. (2020)	Decision-making	Reduce inconsistency	Geometric Consistency Index
Amenta et al. (2020)	Decision-making	Consistency thresholds	Pairwise Comparison Matrices
Han et al. (2020).	Road selection	Apply AHP	AHP
Saaty (2005)	Various applications	Decision-making applications	AHP
Marcarelli & Mancini (2022)	Education	Ranking of school and academic performance	AHP, PROMETHEE
Saaty & Vargas (2012)	Various applications	Models & methods	AHP
Saaty & Vargas (2013)	Various applications	Decision making with ANP	ANP
Moradi & Moradi (2021)	Social	Performance evaluation of a project-based growth and entrepreneurship organization in Iran	BSC, AHP, TOPSIS
Moradi (2022)	Education	Performance evaluation of faculties at the University	BSC, AHP, TOPSIS

2.4.1 Pairwise comparisons

The keystone of the AHP lies in the pairwise comparison of elements at each level of the hierarchy. For each level, experts (decision-makers) compare pairs of elements (criteria or alternatives) with respect to their contribution to the element they are related to at the higher level. These comparisons are made using a 1-9 evaluation scale (see Table 4). For example, if element e_i is moderately more important than e_j , it is assigned a value of 3. These comparisons are organized into a reciprocal matrix M , where $m_{ij} = \frac{1}{m_{ji}}$ for all i, j and $m_{ij} = 1$ for $i = j$.

Table 4
Equivalences of pairwise comparisons

Verbal scale	Numerical scale
Elements are equal	1
One element moderately dominates the other (slightly more important)	3
One element strongly dominates the other (more important)	5
One element very strongly dominates the other (much more important)	7
One element is absolutely dominant (absolutely more important)	9

It is possible to use intermediate values (2, 4, 6, and 8) between two judgments to refine the judgment.

2.4.2 Priority weights

Once the pairwise comparison matrix is constructed, priority weights $w = (w_1, w_2, \dots, w_k)$ are computed by solving the eigenvalue Equation (3):

$$M \cdot w = \lambda_{max} \cdot w \quad (3)$$

where λ_{max} is the largest eigenvalue of M and w is the corresponding eigenvector. The normalized eigenvector w provides the relative importance of each element.

2.4.3 Consistency check

One of the most critical aspects of the AHP is ensuring the consistency of the pairwise comparisons. It seems that the AHP requires decision-makers to be experts in order to be consistent when comparing pairs of elements (criteria or alternatives). Inconsistent judgments should be avoided as they can lead to unreliable results.

To check that judgments made by experts are consistent, Saaty (1980) suggests two measures of consistency:

- Consistency Index (CI) (See Equation (4)):

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (4)$$

where n is the number of elements and λ_{max} the eigenvalue associated with judgment matrix.

- Consistency Ratio (CR) (See Equation (5)):

$$CR = \frac{CI}{RI} \quad (5)$$

where RI (Random Index) is the mean of CI for a number of randomly generated pairwise comparison matrices of size n (Saaty & Vargas, 2013). The matrix may be deemed to have sufficient consistency if the CR is less than 0.1; otherwise, the judgments need to be revised. Table 5 below provides computed RI values corresponding to pairwise comparison matrix sizes.

Table 5
Random indices for the AHP

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.0	0.0	0.5	0.9	1.1	1.2	1.3	1.4	1.4	1.4	1.5	1.4	1.5	1.5	1.5
	0	0	8	0	2	4	2	1	5	9	1	8	6	7	9

2.4.4 Aggregation of weights

The final step of the AHP is to aggregate the weights to obtain the overall priorities for the alternatives. Let us consider the weight w_j for criterion c_j and w_{ij} the weight of alternative a_i with respect to criterion c_j . Equation (6) provides the overall AHP score for alternative a_i :

$$s_i = \sum_{j=1}^m w_j \cdot w_{ij} \quad (6)$$

The alternatives are then ranked based on their final scores s_i . The higher the score, the higher the alternative is ranked.

2.5 Existing extensions of the AHP

The AHP method is often extended or hybridized with other methods or complementary approaches to adapt to different contexts. This trend towards hybridization of the AHP with other methods aims to improve the accuracy and relevance of the method. The greatest difficulty with the AHP is noticeable when it is necessary to make in-depth pairwise comparisons within a hierarchical structure. This difficulty drastically increases when the number of criteria and alternatives becomes significant. In addition, there is the inconsistency of judgments. Table 6 provides a summary of some extensions of the AHP method obtained by hybridizing with other methods. Since there is a very large number of AHP extensions in the literature, we have selected only a few to highlight. The selected AHP extensions were chosen for their hybrid nature, significant influence in literature, and relevance to recent advancements.

Table 6
Extensions of the AHP

Method	Reference	Description	Limitation
Group AHP	Ossadnik et al. (2016)	Group AHP enhances traditional AHP by promoting collective decision-making through the aggregation of individual preferences of group members.	The main limitation of Group AHP is the inconsistency in expert judgments.
DS/AHP	Beynon et al. (2000)	DS/AHP combines the Dempster-Shafer theory of evidence with the AHP.	Complexity in combining evidence and potential computational intensity.
VAHP	Soltanifar & Kamyabi (2024).	VAHP extends traditional AHP by integrating ordinal preferences and voting mechanisms.	Limited by expert availability and potential bias in voting.
AHP-SWARA	Zolfani & Saparauskas (2013)	This method evaluates alternatives by prioritizing expert criteria and initiating the weighting process to determine relative weights.	Its main limitation is the lack of precision observed if the experts relied upon are not well-trained.
SWARA-VAHP	Keršulienė et al. (2010)	Combines Stepwise Weight Assessment Ratio Analysis (SWARA) and VAHP	Relies on well-trained experts to reduce bias.
AHP-BWM	Rezaei (2015)	Merges Analytic Hierarchy Process (AHP) and Best-Worst Method (BWM).	Can be time-consuming.
BWM-VAHP	Mi et al. (2019)	Integrates Best-Worst Method (BWM) with VAHP.	Expert consensus is difficult to achieve.
WM-AHP	Dong et al. (2010)	Combines Weighted Sum Model (WSM) and AHP.	Too sensitive to criteria weights.
BM-AHP	Ishizaka & Labib (2011)	BM-AHP merges Best-Min Method (BM) and AHP.	Requires handling of inconsistencies due to less robust comparisons.
AHP-Gaussian method	Pereira et al. (2023)	Aids in choosing a smart sensor setup for the electric motor powering an escalator in a subway station.	Data must be quantitative.
AHP-TOPSIS-2N	Rodrigues et al. (2024)	Hybrid method combining AHP and TOPSIS with two normalizations; ensures robust rankings.	Requires consistency in pairwise comparisons.
THOR 2	Rodrigues et al. (2024)	Advanced decision-making tool integrating multiple criteria; enhances precision.	Complexity in implementation and computational demands.
Gaussian AHP-TOPSIS-2N	Rodrigues et al. (2024)	Incorporates Gaussian distribution for refined weight calculations; improves accuracy.	Sensitive to parameter selection.
AHP-ELECTRE-TRI	Sagawe et al. (2022)	Combines AHP for criteria weighting with ELECTRE-TRI for classification; enhances decision robustness.	Requires precise threshold settings for accurate classification.

All these derived and/or hybridized methods of the AHP have limitations related to the complexity of their implementation in terms of the number of criteria and alternatives, the time required for pairwise comparisons and consistency analysis, the influence of decision-maker preferences on the results, the lack of rigor in consistency analysis of judgments, the instability of conclusions in case of changes in weights or judgments (sensitivity), the disruption of the order of alternatives based on assigned weights, the complexity of the hierarchical structure of criteria and alternatives, the problem of accurately modeling real situations, the non-compliance with the transitivity of assumptions, the difficulty in obtaining consistent pairwise comparisons, etc. (Munier & Hontoria, 2021; Saaty & Vargas, 2012).

3. Conjoint Analytical Hierarchy Process (CAHP)

Reaching a group decision-making outcome requires amalgamating individual judgments into a unique representative judgment that forms collective choices from individual preferences. Many authors agree with Saaty (2008) in highlighting the significance of the reciprocal property in this process and support the geometric mean as the ideal method for achieving an accurate aggregation of individual judgments (Petrović et al., 2023). Furthermore, Furtado and Johnson (2024) point out that the geometric mean of the columns of a reciprocal matrix is an effective method for deriving priority vectors in the AHP method. Based on this fact, experts can merge their final results, whereas those with differing priorities adjust their judgments by raising them to the power of their respective priorities.

The geometric mean is the ideal central tendency parameter for distributions where outliers can impact the final outcome. Despite its ability to mitigate sensitivity issues in group decision-making, there are still some challenges associated with this approach:

- (1) It is difficult for participants to interpret this parameter compared to the arithmetic mean, which is more intuitive. This can be seen as a drawback that might affect participants' acceptance of the results (Wan Rosanisah & Abdullah, 2016);
- (2) The geometric mean is most effective for log-normally distributed data. If the data do not follow this distribution, this parameter might not be the best choice (Elton & Gruber, 1974);
- (3) It is challenging to ensure that all participants' judgments are consistent with each other when using the geometric mean (Chao, 2008).

In this section, our point of view is quite different. We suggest that when multiple decision-makers have to decide on the importance of each criterion, CA should be used to achieve this. This method evaluates decision-makers' preferences by presenting plan cards with varied attributes. The generated cards were obtained by combining different attribute levels. Decision-makers were then asked to assign scores to the generated cards using a 0-10 evaluation scale to reveal their preferences and trade-offs. We asked participants to rate alternatives' performances on each criterion using the same evaluation scale. Subsequently, we used the arithmetic mean of the performances that decision-makers assigned to each alternative a_i for criterion c_j . The choice of the arithmetic mean is justified by the possibility of zero values when decision-makers assign performances to

alternatives. The geometric mean cannot be computed if any of the performances are zero (de la Cruz & Kreft, 2019). In the next step, a specific formula was used to derive the pairwise comparison matrices for each criterion, and finally, the AHP score was computed for each alternative. Naturally, the alternatives were ranked according to their AHP scores. The higher the score, the higher the alternative is ranked.

3.1 Settlement of criteria weights

In our method, we employed CA to determine the weights assigned to each criterion. Assuming the criteria were already established, each criterion was limited to a manageable number of modalities (levels). Next, we generated plan cards, each representing a fictive or fanciful alternative.

In the next step, decision-makers assigned scores or grades from an evaluation scale to these plan cards. CA processes these evaluations and returns the importance or weights for each criterion. Using CA ensures that the collective preferences of all decision-makers are accurately reflected in the criteria weights. The latter property guarantees the objectivity and robustness of the decision-making process.

3.2 Performance averaging

It is very challenging for the traditional AHP to integrate the judgments of multiple decision-makers, as inconsistencies and subjectivity can affect the final decision outcome (Tavana et al., 2023). To address this issue, we proposed a method that averages the evaluations assigned by each decision-maker to each criterion. We believe this approach is the most effective way to reduce inconsistencies and subjectivity in decision-makers' judgments, ensuring that the final decision reflects a balanced and representative view of all participants. In our view, averaging offers the simplest and most natural path to achieving a globally consistent judgment.

The rationale for this choice lies in the use of a common evaluation scale by decision-makers, with pairwise comparison matrices derived from a unified decision table of alternatives' performances on criteria. Since the arithmetic mean is simple, easy to understand, and accommodates zero values, we agree with Geetha and Raj (2019) that it serves as an effective aggregation parameter. It provides a straightforward and widely accepted method for summarizing individual evaluations into a single collective measure. We computed the arithmetic mean of the performances assigned by all decision-makers for alternative a_i and criterion c_j and obtained the aggregated performance p_{ij}^* . Then, the aggregated performance p_{ij}^* is given by Equation (7):

$$p_{ij}^* = \frac{1}{l} \sum_{k=1}^l p_{ijk} \quad (7)$$

This formula makes it possible to obtain the mean performance score for each alternative on each criterion. The arithmetic mean is probably the most famous central tendency parameter that effectively integrates the evaluations from all decision-makers into a single representative performance score for each alternative and criterion (Ieta et al., 2005).

3.3 Pairwise comparison

It is essential to convert performance vectors $v_j = (p_{ij}^*)$, which represent the performances of alternatives a_i for criteria c_j , into pairwise comparison matrices. This is the only way to effectively perform the AHP. However, this conversion must ensure that the judgments are consistent. The consistency of a judgment is established when the consistency ratio (CR) is 0.10 or less. As maintaining consistency in judgments is so important for the reliability and accuracy of the AHP process, we proposed a formula that converts performance vectors into pairwise comparison matrices with consistent judgments.

Given:

- $v = (p_{ij}^*)$ the vector of aggregated performances of alternative a_i for criteria c_j , where $i = 1, \dots, n$ (alternatives) and $j = 1, \dots, m$ (criteria).
- p_{ij} represents the performance score assigned to alternative a_i for criterion c_j .

To compare alternatives a_i and a_k according to criterion c_j , we used Equation (8) when c_j is to be maximized:

$$f(a_i, a_k) = \begin{cases} Rd\left(\frac{P_{ij} - P_{kj}}{md} + 1\right) & \text{if } P_{ij} > P_{kj} \\ 1 & \text{else} \\ Rd\left(\frac{P_{ij} - P_{kj}}{md} + 1\right) & \end{cases} \quad (8)$$

Where:

- $Rd(x)$ denotes the nearest integer to the real x . We admit that $Rd(4.5) = Rd(4.9) = 5$ but $Rd(1.1) = Rd(1.4) = 1$.
- $md = \frac{\max(v) - \min(v)}{n}$ is the mean deviation, $\max(v)$ and $\min(v)$ denote respectively maximal and the minimal values of P_{ij} .

If criterion c_j were to be minimized, we would rather use Equation (9):

$$f(a_i, a_k) = \begin{cases} Rd\left(\frac{P_{ij} - P_{kj}}{md} + 1\right) & \text{if } P_{ij} < P_{kj} \\ 1 & \text{else} \\ Rd\left(\frac{P_{ij} - P_{kj}}{md} + 1\right) & \end{cases} \quad (9)$$

3.4 Final score computing

In earlier steps of the method, the criteria weights were computed using CA, while the eigenvectors for the pairwise comparison matrices were derived from the AHP. At this stage, we combined these results into a single decision table called "Overall Matrix." Finally, the scores for the alternatives were determined using the weighted sum approach (utility value).

3.5 General algorithm

Most multiple-criteria methods were originally designed for a single decision-maker. Adapting these methods to accommodate multiple decision-makers proved to be a significant challenge, requiring authors to extend single decision-maker methods to multi-decision-maker frameworks. The CAHP is not just an extension of a single decision-maker method (AHP) but also a hybridization of this method with another approach naturally suited for multiple decision-makers (CA). By combining the strengths of the AHP and CA approaches, the CAHP provides a robust solution.

3.6 Computational study on the CAHP

Algorithmic complexity helps analyze algorithms by computing the number of fundamental operations they require before returning the results given the necessary input data. Naturally, the complexity of an algorithm varies with the size of the inputs. Assessing the algorithmic complexity of the CAHP is crucial, as it allows us to be confident using the method even when the size of criteria and alternatives sets increases drastically. Next, we delve into the algorithmic complexity of the CAHP through a step-by-step analysis.

In Step 1, the generation of plan cards is $\mathcal{O}(m)$ for m criteria, the score assignment is $\mathcal{O}(l.p)$ if each of l decision-makers assigns scores to p plan cards, and $\mathcal{O}(l.p)$ is the complexity to process scores. The dominant term for Step 1 is then $\mathcal{O}(l.p)$.

In Step 2, we notice that it takes $\mathcal{O}(n.m.l)$ to collect performance scores for all n alternatives, m criteria, and l decision-makers. The same complexity is required to compute the arithmetic mean of aggregated alternatives' performances on criteria. The overall complexity of the step is then $\mathcal{O}(n.m.l)$.

Step 3 requires $\mathcal{O}(m.n^2)$ to initialize m matrices of size $n \times n$, $\mathcal{O}(m.n^2)$ to compute pairwise comparisons (since each criterion involves $\mathcal{O}(n^2)$ comparisons per criterion, and there are m criteria), and typically, $\mathcal{O}(m.n^3)$ to compute the eigenvectors since the computation of eigenvectors involves matrix operations with complexity $\mathcal{O}(n^3)$ per criterion. The dominant term for Step 3 is $\mathcal{O}(m.n^3)$.

When we analyze Step 4, we can easily check that it takes $\mathcal{O}(m.n)$ for either initializing the overall matrix, or populating that matrix, or computing the final scores. The overall complexity for Step 4 is then $\mathcal{O}(m.n)$.

Step 5 concerns final ranking of alternatives considering their CAHP scores. To rank alternatives from the best to the worst, we need to sort them based on their final scores. The optimal sorting algorithms (e.g., quicksort and mergesort) are $\mathcal{O}(n.\log n)$, with n the number of alternatives. This represents the complexity of Step 5.

The overall complexity is driven by the step with the highest computational cost. In this case, the dominant term is either $\mathcal{O}(m.n^3)$ or $\mathcal{O}(n.m.l)$ from the pairwise comparison matrices step. Therefore, the algorithmic complexity of CAHP can be expressed as:

$$\mathcal{O}(\max(n.m.l, m.n^3))$$

This means that, in the worst case, the complexity scales either cubically with the number of alternatives and linearly with the number of criteria, or linearly with the number of alternatives, criteria, and decision-makers. As shown in Table 7, this level of complexity is reasonable when compared to other aggregation functions designed for multiple decision-makers frameworks.

Table 7
Multiple criteria methods and their complexities

Method	Reference	Complexity	Multiple Decision-Maker
TOPSIS	Hwang & Yoon (1981)	$O(m.n)$	No
ELECTRE (I, II & III)	Roy (1968, 1978a, 1978b)	$O(m.n^2)$	No
PROMETHEE (I, II & III)	Brans & Vincke (1985); Brans & Mareschal (1986, 1994)	$O(m.n^2)$	No
AHP	Saaty (1980)	$O(m^3 + m.n^3)$	No
Group AHP	Ossadnik et al. (2016)	$O(l.m.n^2)$	Yes
DS/AHP	Beynon et al. (2000)	$O(l.m.n^2)$	Yes
AHP/VAHP	Soltanifar & Kamyabi (2024).	$O(l.m.n^2)$	Yes
AHP-SWARA	Zolfani & Saparauskas (2013)	$O(m.n + m^2)$	No
SWARA-VAHP	Keršulienė et al. (2010)	$O(l.n^2.m)$	Yes
AHP-BWM	Rezaei (2015)	$O(m.n + m^2)$	No
BWM-VAHP	Mi et al. (2019)	$O(l.m.n + m^2)$	Yes
WM-AHP	Dong et al. (2010)	$O(l.m.n^2)$	Yes
BM-AHP	Ishizaka & Labib (2011)	$O(m.n^2)$	No
CAHP	Ngoie et al. (2022)	$O(\max(n.m.l, m.l))$	Yes

4. Implementation of the CAHP

In this section, we present two case studies in which the CAHP was applied step by step. The two cases were selected from different fields to illustrate the applicability of the proposed method across various domains.

4.1 Case 1: Ranking stores

In this problem, the goal was to rank the shopping centers from best to worst based on consumer evaluations for each criterion. The selected criteria were:

- Price: Average cost of items sold at the shopping center;
- Quality: Quality of the items sold;
- Distance: Distance between the consumer and the shopping center;
- Welcome: Quality of the welcome provided to customers by the store.

4.1.1 Applying Conjoint Analysis

CA was conducted using SPSS software with the configuration outlined in Table 8. SPSS's ORTHOPLAN tool streamlined the process for respondents by generating only 9

plan cards instead of 54 (see Table 9). Respondents rate each of the 9 plan cards on a 0–10 scale. Based on these ratings, SPSS generated the CA results.

Table 8
Description of the model

Criteria	Number of levels	Modalities	Relation to scores
Price	3	1 – Cheaper	Linear less
		2 – Affordable price	
		3 – Expensive	
Quality	3	1 – Bad quality	Linear more
		2 – Good quality	
		3 – Best quality	
Distance	2	1 – Close	Linear less
		2 – Distant	
Welcome	3	1 – Unwelcoming	Linear more
		2 – Less welcoming	
		3 – Welcoming	

The relationship to scores ‘Linear more’ means that respondents’ preferences increase as the level of a criterion’s modalities rises. Conversely, ‘Linear less’ indicates that as the level increases, preferences decrease. For example, the ‘Price’ criterion is ‘Linear less’ because the cheaper a store’s products, the more consumers prefer it – assuming the stores offer products with identical characteristics across other criteria.

Table 9
Plan cards generated by SPSS

Price	Quality	Distance	Welcome
Expensive	Good quality	Close	Unwelcoming
Expensive	Best quality	Close	Less welcoming
Affordable price	Bad quality	Close	Less welcoming
Affordable price	Best quality	Distant	Unwelcoming
Affordable price	Good quality	Close	Welcoming
Cheaper	Best quality	Close	Welcoming
Cheaper	Bad quality	Close	Unwelcoming
Expensive	Bad quality	Distant	Welcoming
Cheaper	Good quality	Distant	Less welcoming

In Table 10, the CA results indicate that quality (32.371%) emerged as the most important criterion for consumers, followed by price (27.095%), which is often regarded as an index of quality. Welcome (24.890%) and distance (15.644%) ranked third and fourth, respectively.

Table 10
Results of Conjoint Analysis

Criteria	Weight (importance) %	Modalities	Utilities	Std. Error	Inversions
Price	27.095	1 – Cheaper	-0.1	0.138	6
		2 – Affordable price	-0.201	0.276	
		3 – Expensive	-0.301	0.414	
Quality	32.371	1 – Bad quality	0.676	0.138	1
		2 – Good quality	1.352	0.276	
		3 – Best quality	2.028	0.414	
Distance	15.644	1 – Close	-0.682	0.239	7
		2 – Distant	-1.364	0.478	
Welcome	24.890	1 –Unwelcoming	0.286	0.138	5
		2 – Less welcoming	0.572	0.276	
		3 – Welcoming	0.858	0.414	
Constant			4.883	0.586	
		Value	Signification		
Pearson's coefficient r		0.950	0.000		
Kendall's tau (τ)		0.889	0.002		

Individual consumer well-being was gauged by how much happiness an action brings (Igersheim, 2004). Utilities were calculated using the CA method. Low correlation rates indicate inconsistencies, and data with Pearson's $r < 0.7$ or Kendall's $\tau < 0.5$ are rejected (Auty, 1995; Liquet, 2001). Linear models track preference reversals; despite a high number of objects to rank, only a few respondents (19 out of 128 or 14.84%) made inversions. Inversions were more frequent for distance (7) and less frequent for quality (1), reflecting the high importance of quality over other criteria.

Of the 128 respondents, only 19 inversions were found in the responses (only one per respondent). Consumers, therefore, responded with an excellent understanding of the questionnaire. We conclude that this model is very effective, statistically valid ($r=0.95^{***}$; $\tau=0.889^{***}$), and reliably predicts consumers' preferences in terms of choosing different shopping centers based on the retained criteria.

4.1.2 Applying the traditional AHP

After determining the criteria weights through CA, respondents were asked to rate all competing stores on each criterion using a 0–10 evaluation scale. Table 11 presents the average ratings of the stores per criterion along with their weights. For example, the score

of 4.88 obtained by store B for the “Distance” criterion represents the arithmetic mean of the ratings given to B (on ‘Distance’) by all respondents.

Table 11
Unified table of decision

	Price	Quality	Distance	Welcome
Weights (CA)	0.27095	0.32371	0.15644	0.24890
A	3.45	3.78	3.27	3.27
B	5.7	6.06	4.88	5.13
C	7.41	7.09	5.6	6.37
D	6.91	6.92	5.72	6.43
E	5.41	5.01	4.47	4.33
F	6.17	5.64	4.93	5.01

Since all stores are rated from 0 to 10 on each criterion (whether to minimize or maximize), we can consider that all these criteria have been converted into maximization criteria in Table 11. Equation (8) was then used to transform the columns of the decision table into AHP pairwise comparison matrices. Here is a detailed example of this transformation for the “Price” criterion:

$$m = \frac{7.41 - 3.45}{6} = 0.66$$

and

$$f(A, B) = \frac{1}{Rd\left(\frac{5.7 - 3.45}{0.66}\right) + 1} = \frac{1}{4}$$

Since $f(A, B) = \frac{1}{4}$, we can deduce that $f(B, A) = 4$. If we continue comparing the elements pairwise, we will obtain the following matrix:

	A	B	C	D	E	F
A	1	1/4	1/7	1/6	1/4	1/5
B	4	1	1/4	1/3	1	1/2
C	7	4	1	2	4	3
D	6	3	1/2	1	3	2
E	4	1	1/4	1/3	1	1/2
F	5	2	1/3	1/2	2	1

The procedure was repeated for the three other criteria to obtain the remaining AHP pairwise comparison matrices. We can easily check that all pairwise comparison matrices obtained using Equation (8) are consistent. The Value for Money (VFM) vector is the result of multiplying the matrix of alternatives’ scores, also referred to as the Option Preference Matrix (OPM), by the vector of weights obtained from CA. Table 12 shows the AHP eigenvectors along with a relative consistency analysis.

Table 12
Pairwise comparison matrices (performances of alternatives on criteria)

Eigenvectors				
	Price	Quality	Distance	Welcome
A	0.032840687	0.032579130	0.030580695	0.032071061
B	0.094285209	0.150710609	0.128264443	0.132173990
C	0.376152244	0.337447008	0.316747689	0.332184546
D	0.247097151	0.309906574	0.316747689	0.332184546
E	0.094285209	0.066678737	0.079395039	0.063242575
F	0.155339500	0.102677942	0.128264443	0.108143282
Consistency analysis				
λ_{max}	6.13493814	6.16778124	6.13435549	6.16023845
CR	0.02176422*	0.02706149*	0.02167024*	0.02584491*
CI	0.02698763	0.03355625	0.0268711	0.03204769
RI	1.24	1.24	1.24	1.24

* The judgment is consistent since $CR < 0.10$

As shown in Table 13, shopping center C ranks first with 34.34%, while shopping center A ranks last with only 3.22%. The complete ranking is $C > D > B > F > E > A$. We can easily check that AHP and TOPSIS, applied on Table 11, returns the same ranking.

Table 13
Overall CAHP scores for alternatives and final ranking

		Price	Quality	Distance	Welcome	VFM (Final score)	Rank
Alternatives	RVV	0.27095	0.32371	0.15644	0.2489		
	A	0.032840687	0.03257913	0.030580695	0.032071061	0.032210905	6
	B	0.094285209	0.150710609	0.128264444	0.13217399	0.127296904	3
	C	0.376152244	0.337447008	0.316747689	0.332184546	0.343386163	1
	D	0.247097151	0.309906574	0.316747689	0.332184546	0.299503572	2
	E	0.094285209	0.066678737	0.079395039	0.063242575	0.075292788	5
	F	0.1553395	0.102677942	0.128264444	0.108143282	0.122309667	4
Total		1	1	1	1	1	

4.2 Case 2: Ranking virtual labs

This problem involves ranking virtual laboratories (VLs) for teaching physics at the secondary level. The evaluators were teachers who have full mastery of the use of each competing VL. The evaluation criteria were: (1) curriculum alignment, (2) knowledge construction, (3) misconception correction, and (4) usability. The SPSS generated plan cards are given in Table 14.

Table 14
Plan cards for the VL ranking problem

Profile	Curriculum Compliance	Knowledge Building	Misconception correction	Usability
1	Compliant	Partially	Not at all	Very easy
2	Compliant	Not at all	Effectively	Easy
3	Non-compliant	Effectively	Not at all	Easy
4	Non-compliant	Not at all	Partially	Very easy
5	Non-compliant	Partially	Effectively	Difficult
6	Compliant	Not at all	Not at all	Difficult
7	Compliant	Effectively	Effectively	Very easy
8	Compliant	Effectively	Partially	Difficult
9	Compliant	Partially	Partially	Easy

The averaged ratings from judges are given in Table 15.

Table 15
Averaged ratings for the VL ranking problem

VLs	Curriculum Compliance	Knowledge Building	Misconception correction	Usability
VL1	5.2727	7.3636	4.8182	6.6818
VL2	7.5909	7.2727	7.7273	6.7727
VL3	5.8636	7.0909	5.4545	7.2273
VL4	6.0909	7.2727	5.5909	7.3182
VL5	4.7273	7.4091	4.6818	6.8636
VL6	5.5455	6.7727	5.3636	6.9545

The results of the CA in Table 16 indicate the following: ‘misconception correction’ (28.795%) emerged as the most important criterion for the surveyed secondary school teachers, while ‘curriculum compliance’ (26.080%) ranked second. ‘knowledge building’ (24.428%) and ‘Usability’ (20.696%) followed in third and fourth place, respectively.

Table 16
Results of the Conjoint Analysis

Criteria	Weight (importance) %	Modalities	Utilities	Std. Error	Inversions
Curriculum Compliance	26.080	Compliant Non-compliant	-1.917 -3.833	0.211 0.422	0
Knowledge Building	24.428	Effectively Partially Not at all	-0.955 -1.909 -2.864	0.122 0.243 0.365	2
Misconception Correction	28.795	Effectively Partially Not at all	-0.833 -1.667 -2.500	0.122 0.243 0.365	1
Usability	20.696	Very easy Easy Difficult	-0.758 -1.515 -2.273	0.122 0.243 0.365	0
Constant			4.883	0.586	
		Value	Signification		
Pearson's coefficient r		0.991	0.000		
Kendall's tau (τ)		0.889	0.000		

Table 17 shows the AHP eigenvectors along with a relative consistency analysis and the CAHP final ranking.

Table 17
Final ranking with CAHP

	Curriculum Compliance	Knowledge Building	Misconception correction	Usability	CAHP Score	Rank
Weights	0.2608	0.2443	0.2880	0.2070		
VL1	0.0667473222	0.1844037609	0.0566269044	0.0367622245	0.08636788	6
VL2	0.4871833966	0.1119261181	0.5225372541	0.0533426490	0.31590314	1
VL3	0.1403415076	0.0712715546	0.1221610930	0.2991419266	0.15109798	3
VL4	0.1634968811	0.4128873067	0.1307017652	0.4013563009	0.26420037	2
VL5	0.0429944609	0.1844037609	0.0494681575	0.0848516740	0.08806436	5
VL6	0.0992364317	0.0351074990	0.1185048256	0.1245452250	0.09435627	4
Consistency analysis						
λ_{max}	6.1592288	6.1441716	6.1056818	6.2094377		
CR	0.02568*	0.02325*	0.01705*	0.03378*		
CI	0.03185	0.02883	0.02114	0.04189		
RI	1.24	1.24	1.24	1.24		

* The judgment is consistent since $CR < 0.10$

From Table 17, it is clear that the final ranking is $VL2 > VL4 > VL3 > VL6 > VL5 > VL1$. It is easy to verify that the same ranking is obtained using TOPSIS, AHP,

ELECTRE II, and PROMETHEE II – methods specifically designed for ranking problems – based on the same decision table.

5. Discussion

In this article, we introduced the CAHP, an innovative hybrid approach that combines CA and the AHP. This method is original; however, it also inherits advantages and disadvantages from its progenitors.

5.1 Inherited strengths and weaknesses

The CAHP method aims at overcoming the limitations of traditional multi-criteria methods by circumventing the criteria weights set by experts through a survey conducted directly among ordinary consumers whose preferences constitute on average an objective opinion in the criteria weighting process. Since CAHP is a hybridization of the AHP and CA methods, it inherits both the strengths and weaknesses of its progenitors. Due to the reduction in the number of pairwise comparisons achieved by introducing CA to evaluate the weights of criteria, we believe the work of experts was simplified, and inherent inconsistencies were reduced. The use of specific formulas (Equations 8 and 9) transforms tables with evaluations on a 0-10 scale into pairwise comparison tables with consistent judgments. Nonetheless, the weaknesses of the CA and AHP methods (see Table 18) cannot be completely eliminated.

5.2 Innovation and originality of CAHP

The CAHP stands out from the traditional AHP method in how it determines the weight of the criteria. It relies on preferences expressed by a large sample of consumers rather than expert judgments. This approach allows for better representation of real market preferences and reduces potential biases introduced by experts. However gifted they may be, experts remain human and are therefore not immune to errors or subjectivity. Moreover, they are not always available or accessible in all places at all times. Indeed, the traditional AHP, although robust, can be subject to subjective judgments and inconsistencies in setting criterion weights and pairwise comparisons (Aguarón et al., 2020). The CAHP alleviates these issues by simplifying the comparison process and ensuring greater consistency in assessments.

Table 18
Strengths and weaknesses of AHP and CA

Method	Strengths	Weaknesses
AHP	- Breaks down complex problems into a hierarchy of sub-problems; - Accommodates both qualitative and quantitative data; - Includes consistency checks to ensure reliable and consistent pairwise comparison; - Suitable for a wide range of decision-making scenarios.	- Can introduce subjectivity and bias since it relies on experts' judgments for pairwise comparisons; - Can become complex and time-consuming for problems with many criteria and alternatives; - May not be suitable for very large-scale problems; - Ensuring consistency in pairwise comparisons can be challenging.
CA	- Effective in measuring consumer preferences and understanding the trade-offs they are willing to make; - Allows for market simulation and prediction of consumer choices under different scenarios; - Helps identify the relative importance of different product attributes; - Widely used in marketing research, product design, and pricing strategies.	- Conducting conjoint studies can be complex as it requires expertise in survey design and statistical analysis; - May oversimplify consumer preferences by assuming they make trade-offs in a linear and rational manner; - Difficulty increases with the number of attributes and levels; - Some generated plan cards may not exist in real-world scenarios.

5.3 Practical applications and implications

The CAHP has significant practical implications across various sectors, including industry, retail, healthcare, education, and more. As with other traditional methods (Bączkiewicz, 2021), the CAHP method can be used to simulate market scenarios and predict consumer choices, which is particularly useful for marketing and product development strategies. As part of our study, the CAHP was applied in two real-world scenarios. The results it provided were identical to those obtained through AHP, TOPSIS, ELECTRE II, and PROMETHEE II. Additionally, the CAHP is well-suited for multi-decision-maker problems and serves as a less complex extension of AHP.

5.4 Comparison with existing methods

The CAHP offers many advantages over existing methods while achieving the same results. Unlike the traditional AHP, which can become complex and time-consuming as the number of criteria and alternatives increases, the CAHP reduces the number of pairwise comparisons needed while maintaining methodological rigor. Furthermore, the combination of CA and the AHP, as applied in our case study, captures both consumer preferences and the hierarchical structure of decisions, thus providing a more comprehensive and robust approach.

A complexity study of several existing and well-rated methods in the literature has revealed that the CAHP, although designed for multi-decision-maker problems, has a reasonable level of complexity (see Table 7). This strengthens its position as a serious alternative for solving multi-criteria problems with multiple decision-makers, even when the number of criteria, alternatives, and decision-makers increases significantly.

5.5 Limitations and perspective

Even though the CAHP has many advantages, some drawbacks were still observed. Implementing the method may become complex, particularly when designing the survey or analyzing data. Another challenge was preventing inversions in respondent preferences when using CA. In this case, a well-structured classification of criterion modalities could help overcome this issue. Certain biases – such as an asymmetric scale – within the data collection protocol, which may lead to unexpected preferences, are not accounted for in this study. Since the calculation process is tedious, a user-friendly software application could be designed to streamline the implementation of CAHP.

6. Conclusion

This study proposes a novel multi-criteria aggregating function called the Conjoint Analytic Hierarchy Process (CAHP). The proposed method is a hybridization of CA and the the AHP. We illustrated the reliability of the method in the context of consumer preferences for shopping centers. Through the application of the CAHP, we identified quality and price as the most influential factors in consumers' choices. However, consumers have shown a strong preference for high-quality offerings even at a higher cost.

The CAHP represents a significant advancement in the field of multi-criteria decision-making, particularly for scenarios involving multiple decision-makers. It simplifies experts' work and reduces inherent inconsistencies by replacing pairwise comparisons at the criteria level with CA. The CAHP addresses some of the main limitations of traditional methods, such as the limitation to a single decision-maker framework, consistency issues, expert consensus, and sensitivity to criteria weights. Moreover, it can be used in a wide variety of sectors, including business, industry, healthcare, and more.

Future work can focus on developing user-friendly tools, such as software, to facilitate the application of CAHP without requiring extensive training. Additionally, applying CAHP to a broader range of real-world scenarios can help validate its effectiveness and versatility in different contexts. The present study does not investigate decision-makers' behavior while using CAHP, which can be a valuable area of research to understand and mitigate potential biases.

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