BIPOLAR PYTHAGOREAN FUZZY NEUTROSOPHIC SET (BPNS) INTEGRATED WITH AHP EXPRESS (BPNS-AHP EXPRESS) WITH LINGUISTIC VARIABLE: A NEW APPROACH

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ABSTRACT

Decision-making in real-world situations inherently involves uncertainty and insufficient information. To overcome these challenges, decision-makers must adopt techniques that recognize and mitigate the influence of unknown or insufficiently known elements in the decision environment. A single technique often does not address these diverse scenarios, as multiple perspectives may emerge. AHP-express, a simplified form of the Analytic Hierarchy Process (AHP), has recently been introduced; however, the development of a

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new linguistic variable for AHP-express has not been thoroughly investigated. This study initiates the integration of the newly generalized fuzzy set, known as Bipolar Pythagorean Neutrosophic Set (BPNS), with AHP-express (BPNS-AHP express). We develop linguistic variables within the BPNS-AHP express framework using a 7-point linguistic scale, ensuring that BPNS-outlined requirements are met. This articles highlights the six step BPNS-AHP express procedure and demonstrates how it can enhance decision-makers effectiveness across various application domains by providing a more thorough and accurate representation of their thoughts and judgments.

Keywords: BPNS; AHP-express; AHP; multi-criteria decision making

1. Introduction

Multi Criteria Decision Making (MCDM) is widely applied in fields such as project management, environmental planning, finance, and engineering, where decision-makers need to balance multiple conflicting objectives and preferences. In MCDM, decisions are based on the consideration of multiple criteria or attributes. These criteria could be quantitative or qualitative, objective, or subjective, and may have varying levels of importance. Common techniques used in multi-criteria decision-making include AHP, TOPSIS, DEMATEL, and ELECTRE among others (Cruz & Hayos, 2024; Ismail et al., 2023b; Rodzi et al., 2023). The Analytic Hierarchy Process (AHP), developed by Saaty in 1980, is a decision-making methodology designed to handle complex problems involving multiple criteria and alternatives. Saaty's work on the AHP has significantly impacted various fields, including management science, operations research, and decision analysis (Xincheng & Xiang, 2023; Li et al., 2013; Vargas, 2010; Norddin, Ahmad & Yusof, 2015).

The AHP, as a component of MCDM, allows decision-makers to quantify subjective judgments and preferences through pairwise comparisons, leading to more informed and rational decision outcomes (Alam et al., 2023). By structuring decision problems hierarchically, the AHP aids in organizing complex information and relationships, thereby facilitating the decision-making process (Imansyah & Karnaningroem, 2020). The method has been widely adopted in academia and industry for its ability to handle complex decision situations and provide a systematic and transparent decision-making process (Sooksaksun & Chanta, 2023). Hybrid approaches involving the AHP often refer to combining AHP with other decision-making methods or techniques to address specific challenges or enhance the overall decision-making process. These hybrids aim to leverage the strengths of multiple methods to improve the robustness and accuracy of decision outcomes.

Saaty introduced the concept of the Analytic Network Process (ANP), which is the generalization of the AHP method (Saaty, 2005). The ANP is an extension of the AHP that is designed to handle more complex decision problems with non-hierarchical structures and intricate interdependencies. It is suitable for non-hierarchical and more complex decision structures. The ANP allows a more comprehensive modeling of decision scenarios by considering the interactions and dependencies among criteria, making it suitable for situations where criteria influence each other (Namin et al., 2019). By incorporating network structures, the ANP enhances the decision-making process by capturing the complex relationships present in real-world decision contexts. There are many examples of hybrid methods involving the AHP, including AHP-TOPSIS and AHP-Genetic Algorithm. Haktanir (2024) developed a novel intuitionistic Z-AHP (Analytic Hierarchy Process) and Z-TOPSIS (Technique for Order Preference by Similarity to Ideal Solutions) to express uncertain quantities with different degrees of precision, which can be very useful in practical applications. The results show that the proposed methodology can effectively evaluate and rank existing alternatives, considering the uncertainties and complexities of the decision environment.

Aly and Maher (2014) combined the Analytic Hierarchy Process (AHP) with Genetic Algorithms (GA) to address decision-making problems. The AHP and GA are techniques from different domains; the AHP is a multi-criteria decision-making method, while GA is an evolutionary algorithm used for optimization. Artificial Neural Networks (ANN) are computational models inspired by the structure and function of the human brain. In the AHP-ANN hybrid, a neural network is trained to learn the relationship between the prioritized criteria and the decision outcomes based on historical data or expert knowledge. Lin, Fan, and Chen (2022) developed a model for predicting major risk factors and construction quality. The weights of the main risk factors were determined using the AHP to confirm their impact on construction. Meanwhile, ANN was used to extract the characteristics of key risk indicators to forecast the level of construction project quality.

In 1965, Zadeh introduced fuzzy set theory to handle imprecise and uncertain information in decision-making (Zadeh, 1965). Fuzzy sets play a crucial role in decision-making by enabling the modeling of subjective judgments and preferences that may not be easily quantifiable using traditional crisp sets (Samanlıoğlu, 2019). Rodzi and Ahmad (2020) introduce the idea of a new hybrid model, fuzzy parameterized hesitant fuzzy soft sets (FPHFSSs). The benefit of this theory is that the degree of importance of parameters is being provided to HFSSs directly from decision makers. Neutrosophic sets, a concept introduced by Florentin Smarandache in the 1990s, have emerged as a powerful tool for modeling uncertainty in decision-making processes by allowing for the representation of indeterminacy, vagueness, and inconsistency (Nabeeh et al., 2019). By incorporating the concept of indeterminacy, neutrosophic sets offer a more comprehensive framework for capturing the various degrees of truth, falsity, and indeterminacy present in decision contexts (Nabeeh et al., 2019).

Pythagorean fuzzy sets (PFS), introduced by K.T. Atanassov in 2014, represent another extension of fuzzy sets that incorporate the concept of uncertainty with membership, non-membership, and hesitancy degrees (Abdel-Basset et al., 2018). PFS provides a more flexible and expressive framework for handling uncertainty in decision-making by allowing decision-makers to express their preferences in a more granular and detailed manner (Abdel-Basset et al., 2018). This extension enhances the modeling capabilities of traditional fuzzy sets and enables decision-makers to capture the nuances of decision scenarios more effectively (Abdel-Basset et al., 2018). Ismail (2023b) combined neutrosophic fuzzy sets with PFS to model complex systems containing ambiguity, indeterminacy, and uncertainty. Ismail (2023a) also integrated the Pythagorean neutrosophic set with DEMATEL to enhance decision-making problems. The development of bipolar neutrosophic sets further enriches the landscape of decision-making methodologies by combining the concepts of bipolarity and neutrosophic sets to address decision problems with conflicting and contradictory information (Awang & Ali, 2019).

Bipolar neutrosophic sets allow decision-makers to express their preferences in a bipolar manner, considering both positive and negative aspects simultaneously, while also accounting for indeterminacy and uncertainty (Awang & Ali, 2019). Bipolar judging, incorporating both positive and negative features of human perception and cognition, was introduced by Zhang (1994). Deli (2015) integrated the theory of bipolar and neutrosophic sets by considering both positive/negative degrees from bipolar fuzzy sets and truth/indeterminacy/falsity degrees from neutrosophic fuzzy sets within the same modelling framework. This integration enhances the decision-making process by providing a more comprehensive and balanced representation of preferences and considerations (Awang & Ali, 2019). Ahmad et al. (2024) introduced the Bipolar Pythagorean Neutrosophic Set (BPNS), a hybrid set combining fuzzy set theory with bipolar sets, Pythagorean sets, and neutrosophic sets. By incorporating both positive and negative preferences, it is proposed that, BPNS can enhance the expressiveness and flexibility of the model, potentially resulting in more accurate and realistic modelling of complex decision scenarios.

Zadeh's pioneering work laid the foundation for integrating fuzzy logic into decision-making methodologies, leading to the development of Fuzzy Analytic Hierarchy Process (FAHP) as an extension of the AHP (Samanlıoğlu, 2019). The integration of fuzzy set theory with the AHP has resulted in the development of the FAHP, a powerful decision-making tool that addresses the limitations of traditional AHP by accommodating uncertainties and ambiguities in decision processes (Samanlıoğlu, 2019). The integration of fuzzy set theory with the AHP was proposed to address the limitations of AHP in handling subjective judgments and uncertainty. The FAHP enables decision-makers to express preferences using linguistic terms and manage imprecise information in the decision process. The FAHP method can be applied in various fields where decision-making involves imprecise, uncertain, or qualitative information. Some common applications include project selection and evaluation, environmental impact assessment, medical decision-making, and supplier selection (Nahavandi, Homayounfar & Daneshvar, 2023; Akman, Boyaci, & Kurnaz, 2022).

However, Samána et al. (2021) further enhanced FAHP by integrating Interval Type-2 Trapezoidal Fuzzy Sets into the method. While Type-1 Fuzzy Sets (T1FS) typically use crisp numbers to evaluate membership functions, expressing linguistic variables with the degree of membership function can be challenging due to various complexity issues. Thus, Type-2 Fuzzy Sets (T2FS), suggested by Zadeh (1975), offer a solution. Despite its wide applicability, the AHP method suffers from the significant disadvantage of requiring many comparisons to make a decision. Françozo (2023) presents software for the AHP that requires n-1 comparisons, focusing on three problems: (1) many pairwise comparisons with numerous criteria, (2) difficulties in maintaining consistency in judgments, and (3) the absence of software for group decision-making. Leal (2020) introduced a simplified version of the AHP method named AHP-express, that focuses on reducing time constraints by minimizing the number of comparisons. The simplified method calculates priorities for each alternative against a set of criteria with only n-1 comparisons of n alternatives for each criterion, whereas Saaty's original method uses a formula for comparisons of n(n−1)/2 alternatives for each criterion.

AHP-express suggests that the alternative deemed most significant or one of the most important alternatives be used as the baseline for comparison for each priority calculation in each criterion. Notably, the development of new linguistic variables for AHP-express has not been thoroughly explored. Traditional AHP methods, while impactful, often struggle to handle the uncertainty and indeterminacy present in real-world decision-making scenarios. Furthermore, the extensive number of comparisons required by these methods complicates the process, leading to difficulties in maintaining consistency and increasing computational demands. This challenge highlights the need for innovative decision-making models that simplify the process, reduce the number of comparisons, and accurately represent uncertain information. Addressing these challenges is crucial for enhancing the efficiency and effectiveness of decision-making across various fields. Therefore, this study proposes to integrate the concept of the BPNS with the AHP-express method to assess its effectiveness.

The key contributions to the AHP method are as follows:

1. New sets, such as BPNS, are introduced to address decision-making problems, representing uncertain information while considering both positive and negative influences.
2. The BPNS-AHP express method is integrated with the AHP method, which involves n-1 comparisons.
3. Decision-makers can prioritize key criteria to maintain decision quality without compromise.
4. This extended method reduces the number of comparisons, streamlines the process, saves time, and conserves computational resources.

This research aims to enhance efficient decision-making processes. We begin by introducing BPNS, which operates with linguistic variables. Subsequently, we intend to integrate BPNS with AHP using an AHP-express approach. Our objective is to develop a robust tool that merges the adaptability of BPNS with the analytical rigor of AHP. We envision this tool facilitating decision-making for complex problems without consuming excessive time. Our primary objective is to propose a novel model named "BPNS-AHP express" which harnesses the strengths of both methods. We aim to illustrate how this model can be applied in real-life decision scenarios through numerical demonstrations. Through this illustration, we seek to demonstrate that our method simplifies decision-making processes, making them more intuitive and comprehensible, thus guiding decision-makers to make appropriate choices across various contexts.

2. Preliminaries

In this section, the basic definitions related to BPNS are introduced.

Definition 1: (Ahmad et al., 2024). A BPNS with components D in Z is defined as:

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$$D=\left\{\left\langle z,\alpha_D^{+}(z),I_D^{+}(z),\beta_D^{+}(z),\alpha_D^{-}(z),I_D^{-}(z),\beta_D^{-}(z)\right\rangle : z\in Z\right\} \quad (1)$$

Where $\alpha_D^{+}(z),I_D^{+}(z),\beta_D^{+}(z):Z\rightarrow[0,1]$ and $\alpha_D^{-}(z),I_D^{-}(z),\beta_D^{-}(z):Z\rightarrow[-1,0]$

The conditions for all z in Z are satisfied as shown below:

$$0\le \alpha_D^{+}(z)^2+\beta_D^{+}(z)^2\le 1 \quad (2)$$

$$0\le \alpha_D^{-}(z)^2+\beta_D^{-}(z)^2\le 1 \quad (3)$$

$$0\le \alpha_D^{+}(z)^2+I_D^{+}(z)^2+\beta_D^{+}(z)^2\le 2 \quad (4)$$

$$0\le \alpha_D^{-}(z)^2+I_D^{-}(z)^2+\beta_D^{-}(z)^2\le 2 \quad (5)$$

Let $E=\left\{\left\langle z,\alpha_E^{+}(z),I_E^{+}(z),\beta_E^{+}(z),\alpha_E^{-}(z),I_E^{-}(z),\beta_E^{-}(z)\right\rangle : z\in Z\right\}$ Then, $D\subseteq E$ if and only if

$\alpha_D^{+}(z)\le\alpha_E^{+}(z),I_D^{+}(z)\le I_E^{+}(z),\beta_D^{+}(z)\le\beta_E^{+}(z)$ and

$\alpha_D^{-}(z)\ge\alpha_E^{-}(z),I_D^{-}(z)\ge I_E^{-}(z),\beta_D^{-}(z)\ge\beta_E^{-}(z)$ for all z in Z

Also, $D=E$ if and only if

$\alpha_D^{+}(z)=\alpha_E^{+}(z),I_D^{+}(z)=I_E^{+}(z),\beta_D^{+}(z)=\beta_E^{+}(z)$ and

$\alpha_D^{-}(z)=\alpha_E^{-}(z),I_D^{-}(z)=I_E^{-}(z),\beta_D^{-}(z)=\beta_E^{-}(z)$ for all z in Z

The complement of BPNS, D is

$$D^C=\left\{\left\langle z,\beta_D^{+}(z),I_D^{+}(z),\alpha_D^{+}(z),\beta_D^{-}(z),I_D^{-}(z),\alpha_D^{-}(z)\right\rangle\right\} \quad (6)$$

The union D or E are

$$D\cup E=\begin{Bmatrix}\max(\alpha_D^{+},\alpha_E^{+}),\min(I_D^{+},I_E^{+}),\min(\beta_D^{+},\beta_E^{+}),\min(\alpha_D^{-},\alpha_E^{-}),\min(I_D^{-},I_E^{-}),\\ \max(\beta_D^{-},\beta_E^{-})\end{Bmatrix} \quad (7)$$

The intersection D and E are

$$D\cap E=\left\{\min(\alpha_D^{+},\alpha_E^{+}),\max(I_D^{+},I_E^{+}),\max(\beta_D^{+},\beta_E^{+}),\max(\alpha_D^{-},\alpha_E^{-}),\max(I_D^{-},I_E^{-}),\min(\beta_D^{-},\beta_E^{-})\right\} \quad (8)$$

Then, the operations for BPNSs are defined as below:

Definition 2: Let $r = \langle \alpha_r^+, I_r^+, \beta_r^+ \alpha_r^-, I_r^-, \beta_r^- \rangle$ and $s = \langle \alpha_s^+, I_s^+, \beta_s^+, \alpha_s^-, I_s^-, \beta_s^- \rangle$ be two BPNSs and t is a scalar. The following is the definition of the operational rules, which include addition, multiplication, scalar multiplication, and power operations.

$$r \oplus s = \left\langle \begin{matrix} \sqrt{(\alpha_r^+)^2 + (\alpha_s^+)^2 - (\alpha_r^+)^2 \cdot (\alpha_s^+)^2}, I_r^+ I_s^+, \beta_r^+ \beta_s^+, -(\alpha_r^- \cdot \alpha_s^-), \\ -(-I_r^- - I_s^- - I_r^- I_s^-, -\sqrt{(\beta_r^-)^2 + (\beta_s^-)^2 - (\beta_r^-)^2 \cdot (\beta_s^-)^2} \end{matrix} \right\rangle \quad (9)$$

$$r \otimes s = \left\langle \begin{matrix} \alpha_r^+ \cdot \alpha_s^+, I_r^+ + I_s^+ - I_r^+ I_s^+, \sqrt{\left(\beta_r^+\right)^2 + \left(\beta_s^+\right)^2 - \left(\beta_r^+\right)^2 \cdot \left(\beta_s^+\right)^2}, \\ -\sqrt{\left(\alpha_r^-\right)^2 + \left(\alpha_s^-\right)^2 - \left(\alpha_r^-\right)^2 \cdot \left(\alpha_s^-\right)^2}, -\left(I_r^- \cdot I_s^-\right), -\left(\beta_r^- \cdot \beta_s^-\right) \end{matrix} \right\rangle \quad (10)$$

$$t(r) = \left\langle \sqrt{1 - \left(1 - \left(\alpha_r^+\right)^2\right)^t}, \left(I_r^+\right)^t, \left(\beta_r^+\right)^t, -\left(-\alpha_r^-\right)^t, -\left(-I_r^-\right)^t, -\sqrt{1 - \left(1 - \left(-\beta_r^-\right)^2\right)^t} \right\rangle \quad (11)$$

$$(r)^t = \left\langle \left(\alpha_r^+\right)^t, 1 - \left(1 - I_r^+\right)^t, \sqrt{1 - \left(1 - \left(\beta_r^+\right)^2\right)^t}, -\sqrt{1 - \left(1 - \left(-\alpha_r^-\right)^2\right)^t}, -(-I_r^-)^t, -(\beta_r^-)^t \right\rangle \quad (12)$$

3. Methodology

3.1 Construction of linguistic variable

Linguistic variables were first proposed by Zadeh as an effective technique to describe qualitative assessments (Sahoo, 2018). They provide a valuable mechanism for representing subjective assessments that may not be easily quantifiable using traditional crisp sets (Sahoo, 2018). Fuzzy logic, a useful tool for working with linguistic variables, is particularly beneficial when dealing with behavioral data (Valášková et al., 2019). A linguistic variable typically consists of a name, a set of linguistic values, a universal set, a grammar for generating linguistic terms, and a mapping to fuzzy subsets, allowing for a comprehensive representation of qualitative information (Sahoo, 2018). In decision-making processes, linguistic variables are crucial for facilitating reasoning and inference processes. Fuzzy logic controllers utilize linguistic variables to transform inputs into fuzzy variables, establish rules based on linguistic terms, and aggregate rules to derive a unique linguistic result (Zouhair, 2020).

By incorporating linguistic variables into decision-making models, decision-makers can effectively navigate uncertainties and complexities, leading to more informed and accurate decisions (Kebir et al., 2018). This approach allows decision-makers to consider a broader range of factors and perspectives, resulting in a more realistic and nuanced decision-making process (Lanbaran et al., 2020). In conclusion, linguistic variables are crucial in decision-making processes, providing a flexible and intuitive mechanism for representing qualitative assessments and uncertainties. By leveraging linguistic variables, decision-makers can navigate complex decision scenarios more effectively, leading to more informed and reliable decisions. The applications of linguistic variables span diverse domains, from energy management to traffic control systems, highlighting their versatility and significance in modeling and decision-making processes.

In this study, the linguistic variable for BPNS-AHP express is constructed using the BPNS set, allowing for the determination of membership, non-membership, positive membership, negative membership, and indeterminacy. Hashim et al. (2020) developed the linguistic variable within a neutrosophic bipolar environment. Table 1 presents the linguistic variable for neutrosophic bipolar fuzzy sets ranging from "very unimportant" to "very important."

Table 1
Linguistic variable (Hashim et al., 2020)

Score	**Linguistic variable**	**Scale**
1	Very unimportant	[0.1,0.05,0.03, -0.9, -0.45, -0.3]
2	Unimportant	[0.3,0.15,0.1, -0.6.-0.3, -0.2]
3	Medium	[0.55,0.27,0.18, -0.3, -0.15, -0.1]
4	Important	[0.7,0.35,0.23, -0.2, -0.1, -0.06]
5	Very Important	[0.85,0.42,0.28, -0.1, -0.05,-0.03]

The new linguistic variables for BPNS are constructed based on Equation (1). In the classical AHP method, Saaty (1980) introduced a 9-degree scale with four intermediate values. However, in this study, the linguistic variable of BPNS consists of a 7-degree scale, with consideration given to an intermediate value only between “medium to low” and “medium to high” aiming for enhanced precision in evaluation. A 7-degree scale can help reduce the cognitive load on decision-makers. With fewer scale points to consider compared to a 9-degree scale, decision-makers may find it easier to make consistent judgments and maintain focus throughout the decision-making process. Additionally, a 7-degree scale may provide sufficient granularity for capturing relative priorities without overly complicating the decision model. Table 2 presents the linguistic variables for BPNS.

Table 2
Linguistic variable of BPNS

Score	**Linguistic variable**	**Scale BPNS**
1	No Influence	[0.1,0.8,0.9,-0.93,-0.46,-0.31]
2	Low Influence	[0.2,0.7,0.8,-0.45,-0.32,-0.15]
3	Medium Low Influence	[0.35,0.6,0.6,-0.38,-0.19,-0.12]
4	Medium Influence	[0.5,0.4,0.45,-0.32,-0.16,-0.1]
5	Medium High Influence	[0.65,0.3,0.25,-0.21,-0.1,-0.07]
6	High Influence	[0.8,0.2,0.15,-0.12,-0.06,-0.04]
7	Very High Influence	[0.9,0.1,0.1,-0.03,-0.015,-0.01]

The new linguistic variables are verified using all the three conditions stated in Equations (2-5). Based on the definition stated in Equation (1), let

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$W = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$ then,

$$W = \begin{Bmatrix} \langle w_1, 0.1, 0.8, 0.9, -0.93, -0.46, -0.31 \rangle \\ \langle w_2, 0.2, 0.7, 0.8, -0.45, -0.32, -0.15 \rangle \\ \langle w_3, 0.35, 0.6, 0.6, -0.38, -0.19, -0.12 \rangle \\ \langle w_4, 0.5, 0.4, 0.45, -0.32, -0.16, -0.1 \rangle \\ \langle w_5, 0.65, 0.3, 0.25, -0.21, -0.1, -0.07 \rangle \\ \langle w_6, 0.8, 0.2, 0.15, -0.12, -0.06, -0.04 \rangle \\ \langle w_7, 0.9, 0.1, 0.1, -0.03, -0.015, -0.01 \rangle \end{Bmatrix}$$

These sets comply with the conditions $W^+ \in [0,1]$ and $W^- \in [-1,0]$. To provide evidence and demonstrate the validity of the proposed set, conditions in Equations (2-5) are examined as follows:

Let, $\langle w_1, 0.1, 0.8, 0.9, -0.93, -0.46, -0.31 \rangle$ then,

$(\alpha^+)^2 + (\beta^+)^2 \leq 0.1^2 + 0.9^2 = 0.82 \leq 1$ satisfy condition 2

$(\alpha^-)^2 + (\beta^-)^2 \leq (-0.93)^2 + (-0.31)^2 = 0.961 \leq 1$ satisfy condition 3

$(\alpha^+)^2 + (I^+)^2 + (\beta^+)^2 \leq 0.1^2 + 0.8^2 + 0.9^2 = 1.46 \leq 2$ satisfy condition 4

$(\alpha^-)^2 + (I^-)^2 + (\beta^-)^2 \leq (-0.93)^2 + (-0.46)^2 + (-0.31)^2 = 1.23 \leq 2$ satisfy condition 5

Table 3 shows the validation of the set BPNS that has been considered in this study.

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Table 3
Validation of BPNS

Linguistic term	$(\alpha^+)^2 + (\beta^+)^2 \le 1$	$(\alpha^-)^2 + (\beta^-)^2 \le 1$	$(\alpha^+)^2 + (I^+)^2 + (\beta^+)^2 \le 2$	$(\alpha^-)^2 + (I^-)^2 + (\beta^-)^2 \le 2$
No influence	$(0.10)^2 + (0.90)^2 = 0.8200$	$(-0.93)^2 + (-0.31)^2 = 0.9610$	$(0.10)^2 + (0.80)^2 + (0.90)^2 = 1.4600$	$(-0.93)^2 + (-0.46)^2 + (-0.31)^2 = 1.1726$
Low influence	$(0.20)^2 + (0.80)^2 = 0.6800$	$(-0.45)^2 + (-0.15)^2 = 0.2250$	$(0.20)^2 + (0.70)^2 + (0.80)^2 = 1.1700$	$(-0.45)^2 + (-0.32)^2 + (-0.15)^2 = 0.3274$
Medium low influence	$(0.35)^2 + (0.60)^2 = 0.4825$	$(-0.38)^2 + (-0.12)^2 = 0.1588$	$(0.35)^2 + (0.60)^2 + (0.60)^2 = 0.8425$	$(-0.38)^2 + (-0.19)^2 + (-0.12)^2 = 0.1949$
Medium influence	$(0.50)^2 + (0.45)^2 = 0.4525$	$(-0.32)^2 + (-0.10)^2 = 0.1124$	$(0.50)^2 + (0.40)^2 + (0.45)^2 = 0.6125$	$(-0.32)^2 + (-0.16)^2 + (-0.1)^2 = 0.1380$
Medium high influence	$(0.65)^2 + (0.25)^2 = 0.4850$	$(-0.21)^2 + (-0.07)^2 = 0.0490$	$(0.65)^2 + (0.30)^2 + (0.25)^2 = 0.5750$	$(-0.21)^2 + (-0.10)^2 + (-0.07)^2 = 0.0590$
High influence	$(0.80)^2 + (0.15)^2 = 0.6625$	$(-0.12)^2 + (-0.04)^2 = 0.0160$	$(0.80)^2 + (0.20)^2 + (0.15)^2 = 0.7025$	$(-0.12)^2 + (-0.06)^2 + (-0.04)^2 = 0.0196$
Very high influence	$(0.9)^2 + (0.10)^2 = 0.8200$	$(-0.03)^2 + (-0.01)^2 = 0.0010$	$(0.9)^2 + (0.1)^2 + (0.1)^2 = 0.8300$	$(-0.03)^2 + (-0.02)^2 + (-0.01)^2 = 0.0014$

3.2 BPNS-AHP express procedures

The primary purpose of this section is to explain the development of BPNS-AHP express in detail. The proposed method's general structure is depicted in Figure 1.

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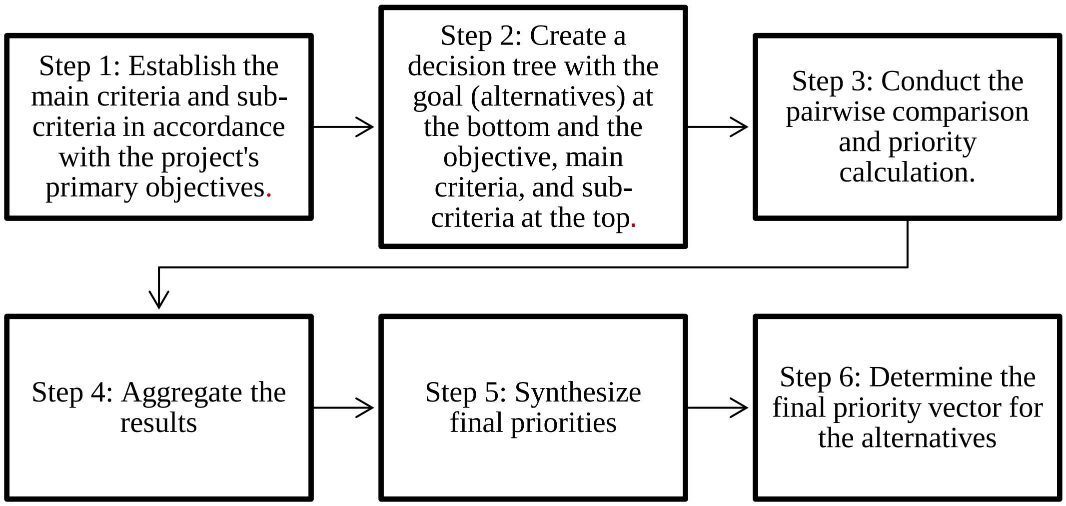

Figure 1 Proposed BPNS-AHP express framework

This integration of BPNS with AHP express (BPNS-AHP express) is completed with the steps enumerated below:

Step 1: Establish the main criteria and sub-criteria in accordance with the project's primary objectives.

Identify and define the main criteria and sub-criteria relevant to the decision-making problem. This foundational step ensures that all significant aspects of the decision are comprehensively considered, providing a structured framework for thorough evaluation.

Step 2: Create a decision tree with the goal (alternatives) at the bottom and the objective, main criteria, and sub-criteria at the top.

Develop a hierarchical structure by creating a decision tree. Place the overall goal at the top, followed by the main criteria, sub-criteria, and alternatives. This organization facilitates a systematic and logical evaluation process, ensuring that all elements of the decision are analyzed in a coherent manner**.**

Step 3: Conduct the pairwise comparison and priority calculation.

Step 3 involves performing pairwise comparisons for each criterion and sub-criterion to determine their relative importance and calculate their priority vectors. This step is essential for establishing the weights of the criteria in the decision-making process.

Create Decision Matrices

For each level of the decision hierarchy, pairwise comparison matrices are constructed. These matrices allow decision-makers to compare each criterion with every other

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criterion to assess their relative importance. The comparison is done using a predefined scale, often utilizing BPNS linguistic variables to capture nuanced judgments about the criterion's importance.

Assign BPNS Linguistic Variables

In each pairwise comparison, BPNS linguistic variables are used to rate the importance of one criterion relative to another. These variables, defined in a previous step, enable the decision-makers to express their preferences in terms that can handle uncertainty and bipolarity. For example, a decision-maker might compare the importance of "cost" versus "quality" using linguistic terms like "very high influence," "medium influence," or "low influence".

Calculate Priority Vectors

Once the pairwise comparisons are made, the next step is to calculate the priority vectors (weights) for each criterion. This involves converting the linguistic judgments into numerical values using the BPNS framework. The priority vector for each criterion is calculated using the following formula:

$$Pr_j = \frac{1}{a_{ij} \sum_k 1/a_{ik}} \tag{13}$$

where a_{ij} is the comparison value between elements i and j, and Pr_j is the priority of the main element j. This calculation provides a set of weights that reflect the relative importance of each criterion based on the pairwise comparisons.

Deneutrosophication Process

In the deneutrosophication process of the BPNS-AHP express method, the parameter represents the level of confidence and ranges from 0 to 1. It adjusts the influence of positive and negative indeterminacy degrees on the crisp value. The formula is:

$$\lambda = \frac{\left[\frac{\alpha^+ + WI^+}{1.5}\right] + \left[1 - \left|\frac{\alpha^- + (1-W)I^-}{1.5}\right|\right]}{\sum_{i=1}^{n} \lambda_n} \tag{14}$$

Where α^+ and α^- are the positive and negative membership degrees, while I^+ and I^- are the positive and negative indeterminancy degrees. When W, high (close to 1), the positive indeterminacy I^+ has a greater impact, emphasizing the positive aspects.

Conversely, a low value $w,$ (close to 0) increases the influence of negative indeterminancy, I^{-} highlighting the negative aspects.

Step 3 is crucial for several reasons. It ensures that the subjective judgments of decision-makers are systematically quantified, allowing for an accurate assessment of the relative importance of each criterion. By using BPNS linguistic variables and the deneutrosophication process, this step also accounts for the inherent uncertainty and bipolarity in human judgments. The resulting priority vectors provide a robust foundation for aggregating the criteria's weights in subsequent steps, ensuring that the final decision reflects the true priorities of the decision-makers.

Step 4: Aggregate the results.

In Step 4, the priority vectors calculated in Step 3 are aggregated to determine the overall priorities of the alternatives. This involves combining the weights of criteria and sub-criteria to provide a comprehensive evaluation.

Formation of PAC Matrix: The PAC matrix is formed by multiplying the priority vectors of sub-criteria (PSC) with the priority vectors of the alternatives $(PASC)$:

$$PAC = PSC * PASC \tag{15}$$

Where PSC represents the priorities of the sub-criteria for each main criterion and $PASC$ represents the priorities of the alternative for each sub-criterion.

This matrix integrates the weighted priorities of sub-criteria under each main criterion, reflecting their influence on the overall decision. For each main criterion, the weighted priorities of its sub-criteria are aggregated to reflect their influence on the overall decision.

Step 5: Synthesize final priorities.

Step 5 involves synthesizing the aggregated priorities to determine the overall priorities of the alternatives. This final step combines the priorities of the main criteria with the PAC matrix to calculate the final priority vector.

Calculate final priorities (PA): The final priority vector PA is calculated by combining the priorities of the main criteria with the PAC matrix:

$$PA = PC * PAC \tag{16}$$

Where PC represents the priorities of the main criteria

This synthesis ensures that the overall decision reflects the importance of each criterion and sub-criterion as judged by the decision-makers. The integration process involves combining the weights of the main criteria (PC) with the aggregated scores from the PAC matrix. This step ensures that the final decision accurately reflects the relative importance of all criteria and sub-criteria.

Step 6: Determine the final priority vector for the alternatives.

In Step 6, the final priority vector for the alternatives is determined by aggregating the weighted priorities from previous steps. This vector provides a clear ranking of the alternatives, indicating which option is most suitable based on the evaluated criteria and sub-criteria. Verification and validation, including consistency checks and optional sensitivity analysis, ensure the reliability and robustness of the decision. This step consolidates all evaluations into a final decision, enabling informed and rational choices while enhancing the transparency and reliability of the decision-making process.

4. Numerical applications

In this section, an example is used to show the applicability of the linguistic variable in decision-making under BPNS-AHP express, which follows the six steps of the proposed framework model previously discussed. Leal (2020) presents a case study from a basic numerical scenario that introduces new concepts using AHP-express. The study considers three levels of hierarchy: two main criteria (C1, C2), four sub-criteria (SC11, SC12, SC21, SC22), and four alternatives (A1, A2, A3, A4). Tables 4, 5, and 6 show the pairwise comparison of each main criteria, sub-criteria, and alternative.

Table 4
Pairwise comparison for the main criteria

	C1	C2
C2	7	1

Table 5
Pairwise comparison for the sub-criteria

C1	SC11	SC12
SC11	3	7

C2	SC21	SC22
SC22	6	1

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Table 6
Pairwise comparison for the alternatives

SC11	A1	A2	A3	A4
A1	1	3	6	7

SC12	A1	A2	A3	A4
A2	5	1	5	7

SC21	A1	A2	A3	A4
A3	7	5	1	3

SC22	A1	A2	A3	A4
A1	1	7	5	9

The proposed BPNS-AHP express with a new linguistic variable was tested using a simple numerical example provided by Leal (2020) as follows:

Step 1: Establish the main criteria and sub-criteria in accordance with the project's primary objectives.

In this study, there are 2 main criteria (Level 1), C1 and C2; 4 sub-criteria (Level 2): SC11, SC12, SC21 and SC22; and 4 alternatives (Level 3): A1, A2, A3 and A4.

Step 2: Create a decision tree with the goal (alternatives) at the bottom and the objective, main criteria, and sub-criteria at the top.

Figure 2 illustrates the decision tree for the identified problems. The objectives of the problem are delineated at the top level, followed by the secondary objective (main criteria) which collectively contributes to the primary objective at the second level. Additionally, the third level is made up of sub-criteria that align with the second objective of the priority structure. Alternatives are then considered at a lower level.

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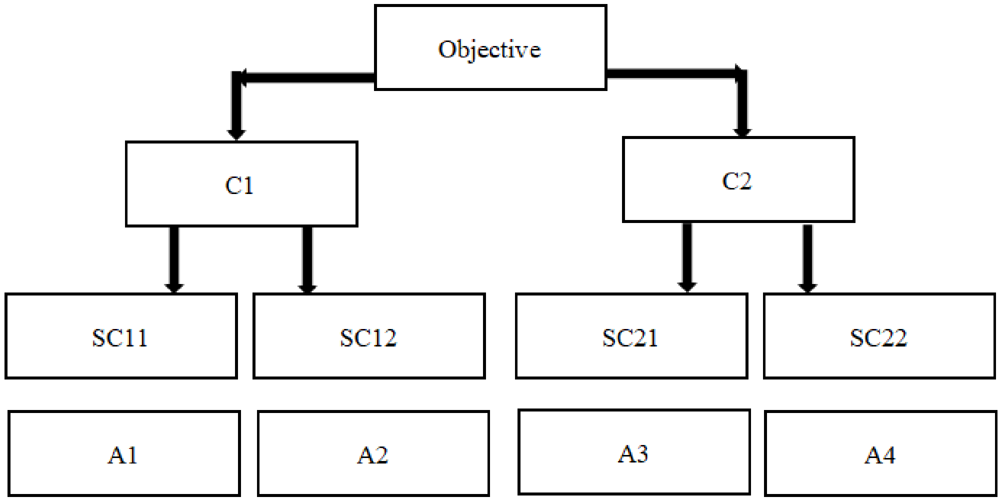

Figure 2 Example of objective hierarchy with two levels of criteria and alternatives in the third level

Step 3: Conduct pairwise comparison and priority calculation.

Create Decision Matrices with BPNS Linguistic Variables

Develop a decision matrix with BPNS linguistic variables for each level by identifying the criteria deemed more significant than others as the primary elements.

Tables 7-9 show the decision matrix of levels 1 to 3 with the proposed BPNS linguistic variable. The primary element is the most important element compared to the other elements. For example, for decision matrix level 1, as in Table 7, C2 is the primary element because C2 is more important than C1.

Table 7
Decision matrix for level 1 incorporating the proposed BPNS linguistic variable

	C1	C2
C2	<0.9,0.1,0.1,-0.03,-0.015,-0.01>	<0.1,0.8,0.9,-0.93,-0.46,-0.31>

In Level 2, within the main criteria C1, the primary element is identified as SC11, while for C2, the primary element is SC22.

Table 8
Decision matrix for level 2 incorporating the proposed BPNS linguistic variable

C1	SC11	SC12
SC11	<0.35,0.6,0.6,-0.38,-0.19,-0.12	<0.9,0.1,0.1,-0.03,-0.015,-0.01>

C2	SC21	SC22
SC22	<0.8,0.2,0.15,-0.12,-0.06,-0.04>	<0.1,0.8,0.9,-0.93,-0.46,-0.31>

In Level 3, the primary elements for the sub-criteria SC11, SC12, SC21, and SC22 are designated as A1, A2, A3, and A1 respectively.

Table 9
Decision matrix for level 3 incorporating the proposed BPNS linguistic variable

SC11	A1	A2	A3	A4
A1	<0.1,0.8,0.9,-0.93,-0.46,-0.31>	<0.2,0.7,0.8,-0.45,-0.32,-0.15>	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	<0.8,0.2,0.15,-0.12,-0.06,-0.04>

SC12	A1	A2	A3	A4
A2	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	(0.1,0.8,0.9,-0.93,-0.46,-0.31))	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	<0.8,0.2,0.15,-0.12,-0.06,-0.04>

SC21	A1	A2	A3	A4
A3	<0.8,0.2,0.15,-0.12,-0.06,-0.04>	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	(0.1,0.8,0.9,-0.93,-0.46,-0.31)	<0.2,0.7,0.8,-0.45,-0.32,-0.15>

SC22	A1	A2	A3	A4
A1	<0.1,0.8,0.9,-0.93,-0.46,-0.31>	<0.8,0.2,0.15,-0.12,-0.06,-0.04>	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	<0.9,0.1,0.1,-0.03,-0.015,-0.01>

Calculate Priority Vectors

Compute the vector of priorities for the criterion using Equation (13).

The results are presented in Tables 10-16.

$$\text{PR11\_A1} = < \frac{10}{39.679}, \frac{1.25}{39.679}, \frac{1.111}{39.679}, \frac{-1.075}{-87.864}, \frac{-2.174}{-87.864}, \frac{-3.226}{-87.864} >$$
$$= < 0.2520, 0.0315, 0.0280, 0.0122, 0.0247, 0.0367 >$$

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Table 10
Calculation of priority for alternatives associated with sub-criterion SC11

SC11	A1	A2	A3	A4
A1	<0.1,0.8,0.9,-0.93,-0.46,-0.31>	<0.2,0.7,0.8,-0.45,-0.32,-0.15>	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	<0.8,0.2,0.15,-0.12,-0.06,-0.04>
1/A1	<10,1.25,1.111,-1.075,-2.174,-3.226>	<5,1.429,1.25,-2.222,-3.125,-6.667>	<2,2.5.2.222,-3.125,0.071,0.134>	<1.25,5,6.667,-8.333,-16.6667,-25>
PR11	<0.252,0.0315,0.028,0.0122,0.0247,0.0367>	<0.126,0.036,0.0315,0.0253,0.0356,0.0759>	<0.0504,0.063,0.056,0.0356,0.0711,0.1128>	<0.0315,0.126,0.168,0.0948,0.1897,0.2845>

$$PR12\_A1 = < \frac{2}{38.722}, \frac{2.5}{38.722}, \frac{2.222}{38.722}, \frac{-3.125}{-95.225}, \frac{-6.25}{-95.225}, \frac{-10}{-95.225} >$$
$$=< 0.0516, 0.065, 0.0574, 0.0328, 0.0656, 0.105 >$$

Table 11
Calculation of priority for alternatives within the sub-criterion SC12

SC12	A1	A2	A3	A4
A2	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	(0.1,0.8,0.9,-0.93,-0.46,-0.31))	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	<0.8,0.2,0.15,-0.12,-0.06,-0.04>
1/A2	<2,2.5,2.222,-3.125,-6.25,-10>	<10,1.25,1.111,-1.075,-2.174,-3.226>	<2,2.5,2.222,-3.125,-6.25,-10>	<1.25,5,6.667,-8.333,-16.667,-25>
PR12	<0.0516,0.065,0.0574,0.0328,0.0656,0.105>	<0.2582,0.0323,0.0287,0.0113,0.0228,0.0339>	<0.0516,0.0645,0.0574,0.0328,0.0656,0.105>	<0.0323,0.1291,0.1721,0.0875,0.175,0.2625>

$$PR21\_A1 = < \frac{1.25}{39.679}, \frac{5}{39.679}, \frac{6.667}{39.679}, \frac{-8.333}{-87.864}, \frac{-16.667}{-87.864}, \frac{-25}{-87.864} >$$
$$=< 0.0315, 0.126, 0.168, 0.0948, 0.1897, 0.2845 >$$

Table 12
Priority calculation for alternatives pertaining to sub-criterion SC21

SC21	A1	A2	A3	A4
A3	<0.8,0.2,0.15,-0.12,-0.06,-0.04>	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	(0.1,0.8,0.9,-0.93,-0.46,-0.31)	<0.2,0.7,0.8,-0.45,-0.32,-0.15>
1/A3	<1.25,5,6.667,-8.333,-16.667,-25>	<2,2.5,2.222,-3.125,-6.25,-10>	<10,1.25,1.111,-1.0753,-2.1739,-3.2258>	<5,1.4286,1.25,-2.222,-3.125,-6.667>
PR21	<0.0315,0.126,0.168,0.0948,0.1897,0.2845>	<0.0504,0.063,0.056,0.0356,0.0711,0.114>	<0.252,0.0315,0.028,0.0122,0.0247,0.0367>	<0.126,0.036,0.0315,0.0253,0.0356,0.0759>

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$$PR22\_A1 = <\frac{10}{53.111},\frac{1.25}{53.111},\frac{1.111}{53.111},\frac{-1.075}{-275.85},\frac{-2.174}{-275.85},\frac{-3.226}{-275.85}>$$
$$=< 0.1882, 0.0235, 0.0209, 0.0039, 0.0079, 0.0117 >$$

Table 13
Calculation of priority for alternatives associated with sub-criterion SC22

SC22	A1	A2	A3	A4
A1	<0.1,0.8,0.9,-0.93,-0.46,-0.31>	<0.8,0.2,0.15,-0.12,-0.06,-0.04>	<0.5,0.4,0.45,-0.32,-0.16,-0.1>	<0.9,0.1,0.1,-0.03,-0.015,-0.01>
1/A1	<10,1.25,1.111,-1.0753,-2.174,-3.2258>	<1.25,5,6.667,-8.333,-16.667,-25>	<2,2.5,2.222,-3.125,-6.25,-10>	<1.111,10,10,-33.333,-66.667,-100>
PR22	<0.1882,0.0235,0.0209,0.0039,0.0079,0.0117>	<0.0235,0.0941,0.1255,0.0302,0.0604,0.0906>	<0.0377,0.047,0.0418,0.0113,0.0227,0.0362>	<0.0209,0.1882,0.1882,0.1208,0.2417,0.3625>

$$PSC1\_SC11 = <\frac{10}{20.04},\frac{1.25}{20.04},\frac{1.111}{20.04},\frac{-1.075}{-18.489},\frac{-2.174}{-18.489},\frac{-3.226}{-18.489}>$$
$$=< 0.499, 0.0624, 0.0554, 0.0582, 0.1176, 0.1745 >$$

Table 14
Priority calculation for sub-criteria within the main criterion C1 (PSC)

C1	SC11	SC12
SC11	<0.1,0.8,0.9,-0.93,-0.46,-0.31>	<0.2,0.7,0.8,-0.45,-0.32,-0.15>
1/SC11	<10,1.25,1.111,-1.0753,-2.174,-3.2258>	<5,1.4286,1.25,-2.222,-3.125,-6.667>
PSC1	<0.499,0.0624,0.0554,0.0582,0.1176,0.1745>	<0.2495,0.0713,0.0624,0.1202,0.169,0.3601>

$$PSC2\_SC21 = <\frac{1.111}{33.472},\frac{10}{33.472},\frac{10}{33.472},\frac{-33.333}{-206.475},\frac{-66.667}{-206.475},\frac{-100}{-206.475}>$$
$$=< 0.0332, 0.299, 0.299, 0.1614, 0.323, 0.4843 >$$

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Table 15
Priority calculation for sub-criteria within main criteria C2 (PSC)

C2	SC21	SC22
SC22	<0.9,0.1,0.1,-0.03,-0.015,-0.01>	<0.1,0.8,0.9,-0.93,-0.46,-0.31>
1/SC22	<1.111,10,10,-33.333,-66.667,-100>	<10,1.25,1.111,-1.0753,-2.174,-3.226>
PSC2	<0.0332,0.299,0.299,0.1614,0.323,0.4843>	<0.2988,0.0373,0.0332,0.0052,0.0105,0.0156>

$$PC\_C1 = < \frac{1.111}{33.472}, \frac{10}{33.472}, \frac{10}{33.472}, \frac{-33.333}{-206.475}, \frac{-66.667}{-206.475}, \frac{-100}{-206.475} >$$
$$= < 0.0332, 0.299, 0.299, 0.1614, 0.323, 0.4843 >$$

Table 16
Priority calculation for main criteria (PC)

	C1	C2
C2	<0.9,0.1,0.1,-0.03,-0.015,-0.01>	<0.1,0.8,0.9,-0.93,-0.46,-0.31>
1/C2	<1.111,10,10,-33.333,-66.667,-100>	<10,1.25,1.111,-1.0753,-2.174,-3.226>
PC	<0.0332,0.299,0.299,0.1614,0.323,0.4843>	<0.2988,0.0373,0.0332,0.0052,0.0105,0.0156>

Deneutrosophication Process

The deneutrosophication process for obtaining crisp values uses Equation (14) to yield a crisp value, as demonstrated in Tables 17-19.

$$PSC11\_C1 = \frac{\frac{(0.499 + (0.0624 * 0.5))}{1.5} + \left(1 - \frac{(0.0582 + (0.1176 * 0.5))}{1.5}\right)}{2.3291} = 0.5476$$

Table 17
Crisp values of priority for the sub-criteria within each criterion (PSC)

	SC11	SC12	SC21	SC22
C1	0.5476	0.4524	0	0
C2	0	0	0.4294	0.5706

$$PSC11\_A1 = \frac{\frac{(0.252 + (0.0315 * 0.5))}{1.5} + \left(1 - \frac{(0.0122 + (0.0247 * 0.5))}{1.5}\right)}{3.1058} = 0.3742$$

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Table 18
Crisp values of priority for the alternatives within each sub-criterion (PASC)

	A1	A2	A3	A4
SC11	0.3742	0.3436	0.3243	0.3015
SC12	0.2445	0.2821	0.2445	0.2290
SC21	0.2244	0.2413	0.2785	0.2558
SC22	0.2768	0.2470	0.2516	0.2246

$$PC\_C1 = \frac{\frac{(0.0332 + (0.2988 * 0.5))}{1.5} + \left(1 - \frac{(0.1614 + (0.3229 * 0.5))}{1.5}\right)}{2.111} = 0.4294$$

Table 19
Crisp values of priority of the main criteria

C1	C2
0.4294	0.5706

Step 4: Aggregate the results.

Table 20 shows the priority of PSC versus PASC. The priorities of the alternatives of each main criterion (PAC) were obtained using Equation (15). The PAC results are displayed in Table 21.

Table 20
Priority of the sub-criteria for each criterion (PSC) versus priority of the alternative for each sub-criterion (PASC)

PSC				PASC			
SC11	SC12	SC21	SC22	A1	A2	A3	A4
0.5476	0.4524	0	0	0.3742	0.3436	0.3243	0.3015
0	0	0.4294	0.5706	0.2445	0.2821	0.2445	0.2290
				0.2244	0.2413	0.2785	0.2558
				0.2768	0.2470	0.2516	0.2246

$$PAC\_C1A1 = (0.5476 * 0.3742) + (0.4524 * 0.2445) + (0 * 0.2244) + (0 * 0.2768) = 0.3155$$

Table 21
Priorities of the alternatives in each criterion (PAC)

	A1	A2	A3	A4
C1	0.3155	0.3099	0.2882	0.2687
C2	0.2543	0.2446	0.2632	0.2380

Step 5: Synthesize final priorities.

Create a schematic representation of the matrix calculation for the final priorities. Table 22 illustrates the matrix multiplication schema used to derive the final priorities. The priorities of each criterion have been calculated using Equation (16). The final priority for the alternatives is depicted in Table 23.

Table 22
Schema of the matrix multiplication

PC		PAC			
C1	C2	A1	A2	A3	A4
0.4294	0.5706	0.3155	0.3099	0.2882	0.2687
		0.2543	0.2446	0.2632	0.2380

$$PA\_A1 = (0.4294 * 0.3155) + (0.5706 * 0.2543) = 0.2806$$

Table 23
Final priorities of the alternatives

	A1	A2	A3	A4
PA	0.2806	0.2540	0.1725	0.1519

Step 6: Determination of final priority vector for the alternatives

In Step 6, the final priority vector for the alternatives is determined by aggregating the weighted priorities from the previous steps. Table 24 shows the final ranking of the alternatives.

Table 24
Ranking of the alternatives

	A1	A2	A3	A4
PA	0.2806	0.2540	0.1725	0.1519
RANK	1	2	3	4

The findings of the decision-making problem indicate that the priority of A1 is 0.2806, which is higher than the priorities of the other alternatives. Following A1, the priorities of A2 and A3 are lower, with A4 having the lowest priority.

5. Discussion

The suggested method was validated, and its accuracy was observed through comparative analysis and sensitivity analysis to see the robustness and reliability of the BPNS-AHP express method.

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5.1 Comparative analysis

The comparative results between the AHP-express and BPNS-AHP express approaches are presented in Table 25.

Table 25
Comparative analysis

Method	Priorities	Ranking
AHP-express	0.108<0.159<0.245<0.488	A4<A2<A3<A1
BPNS-AHP express	0.1518<0.1725<0.2540<0.2806	A4<A3<A2<A1

The ranking order for BPNS-AHP express and AHP-express exhibits significant agreement for rank 1 and rank 4 but differs for ranks 2 and 3. This differentiation arises due to the use of crisp numbers in AHP-express, leading to a substantial gap between the values of linguistic variables, thereby impacting the ranking. A crisp number represents a precise and definite value without any uncertainty or vagueness. BPNS-AHP express provides a more accurate value by representing a range of possible values with varying degrees of membership. The conventional method, utilizing crisp numbers, fails to adequately address uncertainty, whereas the proposed model considers elements that handle indeterminacy, thereby enhancing differentiation between positive and negative uncertainties.

Furthermore, BPNS-AHP express introduces a novel linguistic variable that encompasses degrees of truth, falsity, and indeterminacy, as well as positive and negative influences. This empowers decision-makers to navigate intricate decision scenarios with enhanced precision and flexibility. In real-life situations, certain complex problems may remain unsolved when employing conventional linguistic variables, often due to overlooked elements. Therefore, the development of a new linguistic variable is crucial for accurately representing the opinions of decision-makers.

5.2 Sensitivity analysis

The sensitivity analysis helps explain how changes in the confidence level can impact the final decision. By adjusting the parameter, W within its range, the robustness of the decision-making process is tested. If the rankings of the alternatives remain consistent across different values of W, it indicates a robust and reliable decision model. This step is crucial in validating the decision-making framework and ensuring that the chosen alternative remains the best option under varying levels of confidence and uncertainty. These adjustments and verification enhance the credibility of the decision-making process, providing decision-makers with a thorough understanding of how changes in key parameters affect the outcome.

A sensitivity analysis was conducted to ascertain the impact of various model parameters and inputs on the ultimate decision outcome. These characteristics were schematically adjusted to see how adjustments affected the alternatives' rankings or preferences. Parameter W, where {W = 0.1, 0.3, 0.5, 0.7, 0.9, 1}, was regarded as a variable. Table 26 displays the findings of the sensitivity study and Figure 3 shows the sensitivity analysis graph.

Table 26
Sensitivity analysis

			PA		
W	A1	A2	A3	A4	RANK
0.1	0.3134	0.3079	0.2877	0.2520	A1>A2>A3>A4
0.3	0.3160	0.3101	0.2881	0.2609	A1>A2>A3>A4
0.5	**0.2806**	**0.2540**	**0.1725**	**0.1519**	**A1>A2>A3>A4**
0.7	0.3103	0.3045	0.2863	0.2738	A1>A2>A3>A4
0.9	0.3071	0.3012	0.2852	0.2795	A1>A2>A3>A4
1.0	0.3056	0.2996	0.2847	0.2823	A1>A2>A3>A4

Table 26 presents the results of the sensitivity analysis to show the impact of varying the confidence level parameter W on the final priority rankings of alternatives A1, A2, A3, and A4. The parameter W ranges from 0.1 to 1.0, representing different levels of confidence in the decision-making process. As W changes, the priorities for each alternative are recalculated. Despite these changes, the ranking order of the alternatives remains consistent, with A1 consistently ranked highest, followed by A2, A3, and A4. For instance, at $W = 0.1$, the priorities are 0.3134 for A1, 0.3079 for A2, 0.2877 for A3, and 0.2520 for A4, resulting in the ranking A1 > A2 > A3 > A4. This pattern holds true across all tested values of W, indicating that A1 is the most preferred alternative under varying confidence levels. This consistent ranking demonstrates the robustness and stability of the decision model, suggesting that the final decision is reliable and less sensitive to changes in the confidence parameter W.

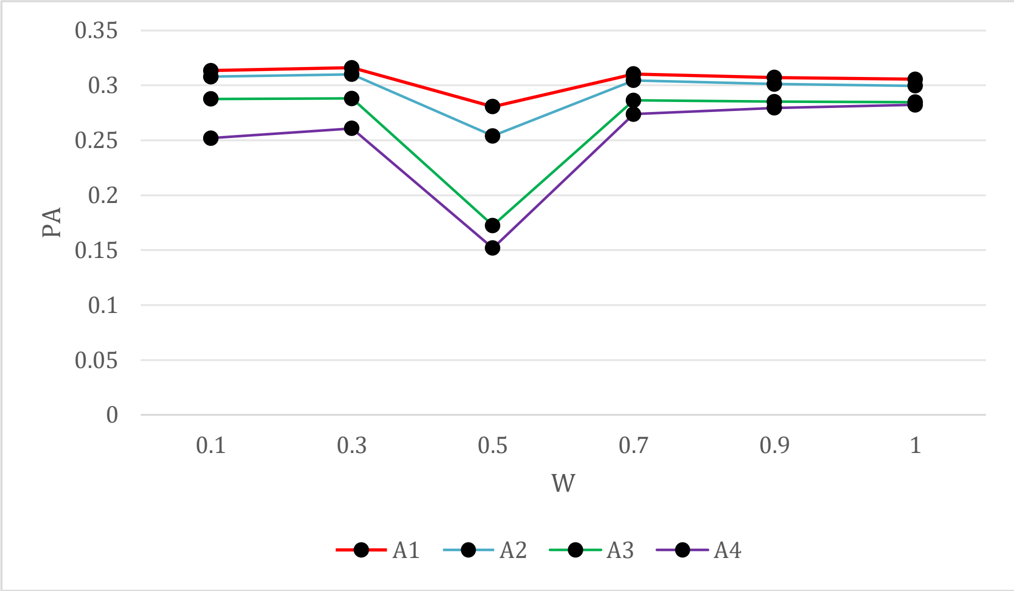

Figure 3 Graph of sensitivity analysis

Figure 3 shows how the priorities PA of alternatives A1, A2, A3, and A4 change with varying confidence levels, W from 0.1 to 1.0. The graph indicates that A1 consistently has the highest priority, followed by A2, A3, and A4, regardless of the value of W. While the priority values fluctuate slightly, the overall ranking of the alternatives remains stable (A1 > A2 > A3 > A4). This stability suggests that the decision model is robust and reliable, as the preferred alternative A1 remains consistent across different levels of confidence.

6. Managerial implications

The integration of the BPNS with AHP-express has significant implications for decision-making processes. By enhancing the accuracy and reliability of these processes, this method allows for more informed and balanced decisions through comprehensive evaluations that consider both positive and negative influences. It effectively handles uncertainty and imprecise information, particularly in complex scenarios like strategic planning, project evaluation, and risk assessment. The simplified AHP-express method reduces the number of pairwise comparisons required, saving time and computational resources, thus allowing a focus on critical decision-making aspects and quicker responses to changing business environments. The structured framework of BPNS-AHP express ensures consistency in judgments across different stages, mitigating individual biases and enhancing decision quality.

Moreover, the method's scalability and flexibility make it adaptable to various decision-making scenarios across different industries, allowing for customization of criteria and sub-criteria according to specific organizational needs. This adaptability makes the method highly applicable in diverse contexts. The use of linguistic variables that align with natural expressions of preferences and judgments enhances stakeholder engagement, fostering active participation in the decision-making process and securing greater buy-in and support for final decisions. By clearly defining and prioritizing criteria, the method helps align decisions with organizational strategic objectives, ensuring that decisions contribute to achieving long-term goals and enhancing overall performance.

Additionally, the BPNS-AHP express method includes a robust sensitivity analysis component, allowing an understanding of the impact of varying parameters on the final decision. This analysis ensures the robustness and stability of decisions, confirming that the chosen alternatives remain optimal under different scenarios. In summary, integrating BPNS with AHP-express offers a powerful tool to improve decision-making quality, efficiency, and flexibility.

7. Limitations and future research directions

While the integration of BPNS with AHP-express offers significant improvements in decision-making processes, several limitations should be noted. First, the method's reliance on linguistic variables and fuzzy logic, while beneficial for handling uncertainty, may introduce subjectivity in the interpretation of these variables. This subjectivity can potentially affect the consistency and reliability of the results. Additionally, the accuracy of the BPNS-AHP express method depends on the quality and comprehensiveness of the input data. Inadequate or biased data can lead to suboptimal decision outcomes.

Another limitation is the computational complexity involved in the deneutrosophication process and the handling of large datasets. As the number of criteria and alternatives increases, the method may require significant computational resources and time, which could be a constraint in real-time decision-making scenarios. Furthermore, while the method is adaptable to various decision-making contexts, its effectiveness in extremely dynamic environments with rapidly changing variables has not been fully tested.

Future research should focus on addressing these limitations by developing more refined techniques for interpreting and standardizing linguistic variables to reduce subjectivity. Additionally, exploring methods to streamline the computational processes involved in BPNS-AHP express will be crucial for enhancing its applicability in real-time and large-scale decision-making scenarios. Further studies should also investigate the integration of BPNS-AHP express with other advanced decision-making frameworks and technologies, such as artificial intelligence and machine learning, to enhance its robustness and adaptability in dynamic environments.

8. Conclusion

The integration of the BPNS with AHP-express (BPNS-AHP express) provides a powerful and flexible approach to multi-criteria decision-making. By incorporating both positive and negative influences and utilizing fuzzy logic, this method addresses the inherent uncertainty and imprecision in real-world decision scenarios. The simplified AHP-express reduces the number of required comparisons, saving time and computational resources, while maintaining the accuracy and reliability of the decision-making process. The method's adaptability allows for its application across various industries and decision-making contexts, from strategic planning and project evaluation

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to risk assessment. By enhancing stakeholder engagement using natural linguistic expressions and aligning decisions with organizational strategic objectives, BPNS-AHP express ensures that decisions are both comprehensive and aligned with long-term goals.

Despite these advantages, the method's reliance on linguistic variables and fuzzy logic requires a high degree of computation. Future research should aim to refine the interpretation of linguistic variables, reduce computational complexity, and explore the integration with other advanced decision-making frameworks. In conclusion, BPNS-AHP express represents a significant progress in the field of multi-criteria decision-making, offering a robust, efficient, and adaptable tool for handling complex decision scenarios. Its ability to incorporate uncertainty and provide a structured framework for evaluation makes it an asset for enhancing decision quality and effectiveness in various organizational contexts. Further research and development will continue to improve its applicability and performance, ensuring its relevance and utility in an ever-evolving decision-making landscape.

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